Slide 1 **Calculus I Announcements**

- Read section 6.1 and 6.2
- Do the homework from section 6.1

Slide 2 **The Definite Integral (Review)**

1. The **indefinite integral** is written $\int f(x)\,dx$ and means to take the anti-derivative.
2. The **definite integral** is written $\int_a^b f(t)\,dt$ and is defined by taking the limit Riemann Sums as the partition size gets small and the number of rectangles goes to infinity. The function $f(t)$ is called the integrand and t is a "dummy variable".
3. The **Fundamental Theorem of Calculus** links the two together and says that differentiation and integration are inverse processes.

Slide 3 **Finding Areas and Volumes (5.6,6.1)**

Areas and volumes of curved regions can be found by using the definite integral. The central ideas are:

****Integrate height to find area****

****Integrate area to find volume****

- Explain why these are true by interpreting the definite integral notation.

For the region that lies between $y = x^2$ and $y = 8 - x^2$, set up, but do not evaluate integrals to:

1. Find its area.
2. Find the volume of the solid that lies above the region where each cross section perpendicular to the x-axis is a square whose base runs from the lower curve to the upper curve.

Slide 4 **iClicker (6.1)**

Question Consider the same region that lies between $y = x^2$ and $y = 8 - x^2$. Set up, but do not evaluate an integral for the volume of the solid that lies above the region where each cross section perpendicular to the x-axis is a half disk whose diameter runs from the lower curve to the upper curve.

A. $\int_{-2}^{2} (8 - 2x^2)\, dx$

B. $\int_{-2}^{2} (8 - 2x^2)^2\, dx$

C. $\int_{0}^{2} 2\pi(8 - 2x^2)\, dx$

D. $\int_{0}^{2} 2\pi(8 - 2x^2)^2\, dx$

E. None of the above

Answer to Question Consider the same region that lies between $y = x^2$ and $y = 8 - x^2$. Set up, but do not evaluate an integral for the volume of the solid that lies above the region where each cross section perpendicular to the x-axis
is a half disk whose diameter runs from the lower curve to the upper curve.

A. $\int_{-2}^{2} (8 - 2x^2)\, dx$

B. $\int_{-2}^{2} (8 - 2x^2)^2\, dx$

C. $\int_{0}^{2} 2\pi(8 - 2x^2)\, dx$

D. $\int_{0}^{2} 2\pi(8 - 2x^2)^2\, dx$

E. None of the above is the correct answer.

Slide 5 **Volumes of Solids of Revolution (6.1)**

Disk Method We use this idea to find volumes of solids of revolutions by integrating areas of disks as in the following example.

1. Sketch a picture of the solid of the region under the curve $y = x^2$, above the x-axis and between $x = 0$ and $x = 1$.
2. Visualize and/or sketch a picture of the this region revolved about the x-axis.
3. Visualize and/or sketch a cross sectional disk formed from the revolution.
4. Write down the radius of the disk as a function of x
5. Write down the Area of the disk as a function of x
6. Write down a typical Riemann sum for an approximation to the volume as the sum of volumes of disks
7. Integrate the area times dx to get the volume.

Slide 6 **iClicker (6.1)**

Question Find the volume of the solid of revolution formed by revolving the region bounded by $y = \sqrt{x}$, the x-axis, $x = 1$ and $x = 3$ about the x-axis.

A. $\pi/3$

B. $4\pi/3$

C. $\pi/2$

D. $3\pi/2$

E. None of the above

Answer to Question Find the volume of the solid of revolution formed by revolving the region bounded by $y = \sqrt{x}$, the x-axis, $x = 1$ and $x = 3$ about the x-axis.

A. $\pi/3$

B. $4\pi/3$

C. $\pi/2$

D. $3\pi/2$

E. None of the above   is the correct answer.

Slide 7 **Washer Method: Horizontal Axes (6.1)**

To find the volume of a region revolved about an axes parallel to the x axis, try the following steps.

1. Sketch the region and find the limits of integration.
2. Try to visualize the revolution and the solid.
3. Find the inner radius $r_1(x)$ as a function of x by finding the vertical distance from the inner function to the axis of revolution.
4. Find the outer radius $r_2(x)$ as a function of x by finding the vertical distance from the outer function to the axis of revolution.
5. Use the formula

$$V = \int_a^b \pi((r_2(x))^2 - (r_1(x))^2)\, dx$$

Example
Set up (but do not integrate) an integral for the volume of the solid of revolution formed by revolving the region bounded by $y = x^2$ and $y = 2$ around the line $y = 5$.

Slide 8 **iClicker (6.1)**

Question What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by $y = \sqrt{x}$, and $y = x^2$ about the x-axis.

A. $\displaystyle\int_0^1 \pi \left(\sqrt{x} - x^2\right)^2 \, dx$

B. $\displaystyle\int_0^1 \pi \left(\left(\sqrt{x}\right)^2 - \left(x^2\right)^2\right) \, dx$

C. $\displaystyle\int_0^1 \pi \left(\left(x^2\right)^2 - \left(\sqrt{x}\right)^2\right) \, dx$

D. $\displaystyle\int_0^1 \pi \left(x^2 - \sqrt{x}\right)^2 \, dx$

E. None of the above

Answer to Question What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by $y = \sqrt{x}$, and $y = x^2$ about the x-axis.

A. $\int_0^1 \pi \left(\sqrt{x} - x^2\right)^2 \, dx$

B. $\int_0^1 \pi \left(\left(\sqrt{x}\right)^2 - \left(x^2\right)^2\right) \, dx$ **is the correct answer.**

C. $\int_0^1 \pi \left(\left(x^2\right)^2 - \left(\sqrt{x}\right)^2\right) \, dx$

D. $\int_0^1 \pi \left(x^2 - \sqrt{x}\right)^2 \, dx$

E. None of the above

Slide 9 **iClicker (6.1)**

Question What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by $y = x$, and $y = x^2$ about the line $y = -2$.

A. $\int_0^1 \pi \left(\left(x^2 - 2\right)^2 - (x-2)^2 \right) \, dx$

B. $\int_0^1 \pi \left(\left(x^2 + 2\right)^2 - (x+2)^2 \right) \, dx$

C. $\int_0^1 \pi \left((x-2)^2 - \left(x^2 - 2\right)^2 \right) \, dx$

D. $\int_0^1 \pi \left((x+2)^2 - \left(x^2 + 2\right)^2 \right) \, dx$

E. None of the above

Answer to Question What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by $y = x$, and $y = x^2$ about the line $y = -2$.

A. $\displaystyle\int_0^1 \pi \left(\left(x^2 - 2\right)^2 - (x-2)^2 \right) \, dx$

B. $\displaystyle\int_0^1 \pi \left(\left(x^2 + 2\right)^2 - (x+2)^2 \right) \, dx$

C. $\displaystyle\int_0^1 \pi \left((x-2)^2 - \left(x^2 - 2\right)^2 \right) \, dx$

D. $\displaystyle\int_0^1 \pi \left((x+2)^2 - \left(x^2 + 2\right)^2 \right) \, dx$ **is the correct answer.**

E. None of the above

Slide 10 **Vertical Axes (6.1)**

We can also revolve regions about axes parallel to the y-axis. The procedure is the same, except that we integrate with respect to y and measure radii horizontally (instead of vertically).

Example

Set up (but do not evaluate) an integral to find the volume of the solid of revolution formed by revolving the region bounded by $y = x$ and $y = \sqrt{x}$ about the line $x = 2$.

Slide 11 **Method of Shells (6.2)**

An alternate method to find area is using shells instead of washers or disks. It is sometimes easier.

1. Sketch a picture of the region bounded the curve $y = \sqrt{x}$, the y-axis and the line $y = \sqrt{2}$
2. Visualize/and or sketch a picture of the this region revolved about the x-axis.
3. Sketch the height and radius of a shell formed from the revolution.
4. Write down the radius of the shell as a function of y
5. Write down the height of the shell as a function of y
6. Write down the area of the shell as a function of y
7. Write down a typical Riemann sum for an approximation to the volume as the sum of volumes of shells.
8. Integrate the area times dy to get the volume.