Slide 1 **Calculus I Announcements**

- You need to do ALL the homework problems from the course web page for the sections we cover in class.
- We will do sections 2.2 and 2.4 today and come back to section 2.3 next time.
- Read sections 2.2 and 2.4 and try the problems from these two sections.

Slide 2 **Rates of Change (2.1)**

Let's try to determine how fast the function given by $f(x) = 5 - x^2$ is rising or falling near $x = 1$

- Graph the function and plot the point on the graph at $x = 1$
- Find the slope of the line between the points $(1, f(1))$ and $(2, f(2))$
- Find the slope of the line between the points $(1, f(1))$ and $(1 + h, f(1 + h))$ and illustrate.
- What happens to this slope as h gets close to zero?
- Find the tangent line to the graph at the point $(1, 4)$

The slope of a function is determined from the slope of the tangent line, or the line that just "kisses" the curve at the given point. The procedure above shows how we find the tangent line.

Slide 3 **Limits (2.2)**

To carry out the process on the previous slide, we had to let h get close to zero. This is the process of *taking limits* which is what chapter 2 is about.

We will use the notation:

$$\lim_{x \to a} f(x)$$

Read aloud this says *the limit as x goes to a of f of x.*

We write that the limit as x goes to a of $f(x)$ is equal to L as

$$\lim_{x \to a} f(x) = L$$

This means that
As x gets close to a, but with $x \neq a$, the value of $f(x)$ gets close to L

Use this description to find the following limits or show they don't exist

1. $\lim_{x \to -2} x^2$
2. $\lim_{x \to 3} \dfrac{(x-3)(x+3)}{(x-3)}$
3. $\lim_{x \to 0} \dfrac{\sin(x)}{x}$
4. $\lim_{x \to 0} \dfrac{|x|}{x}$

Slide 4 **Techniques for Finding Limits (2.2)**

We used analytic, conceptual methods as well as numerical and tabular methods for finding the limits on the previous page. Section 2.2 is about developing a set of general techniques that begin to formalize this.

Here are some simple rules about limits. Write out some examples of these.

1. $\lim_{x \to a} c = c$ where c is constant.
2. $\lim_{x \to a} x = a$
3. $\lim_{x \to a} x^n = a^n$ where n is positive.
4. $\lim_{x \to a} P(x) = P(a)$ if P is a polynomial
5. $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ for $a \geq 0$.
6. $\lim_{x \to a} \ln(x) = \ln(a)$ for $a > 0$.
7. $\lim_{x \to a} e^x = e^a$
8. $\lim_{x \to a} c^x = c^a$ for a constant $c > 0$.
9. $\lim_{x \to a} \sin(x) = \sin(a)$ and similarly for cosine.
10. $\lim_{x \to a} \tan(x) = \tan(a)$ as long as $\tan(a)$ is defined (not infinity). Similarly for other trig functions.

Slide 5 **Limits: More Rules (2.2)**

If f and g are functions and $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} g(x)$ exists then

1. $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a}(f(x)\cdot g(x)) = \left(\lim_{x\to a} f(x)\right)\cdot\left(\lim_{x\to a} g(x)\right)$
3. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ provided that $\lim_{x\to a} g(x) \neq 0$.
4. $\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$ provided that $\lim_{x\to L} f(x)$ exists where $L = \lim_{x\to a} g(x)$.

Find the limits

$$\lim_{x\to\pi/6} x^2 + 5x\sin(x) \qquad \lim_{x\to 2} \frac{e^x+3}{x+4} \qquad \lim_{x\to 2} \sin(\cos(e^x))$$

Slide 6 **iClicker (2.2)**

Question $\lim_{s \to 4} 3(2s-1)/s$ equals

A. $3(2s-1)/s$

B. $3(8-1)/s$

C. $3(2s-1)/4$

D. $7/4$

E. None of the above

Answer to Question $\lim_{s\to 4} 3(2s-1)/s$ equals

A. $3(2s-1)/s$

B. $3(8-1)/s$

C. $3(2s-1)/4$

D. $7/4$

E. None of the above **is the correct answer.**

Slide 7 **Limits: Simplification Techniques (2.2)**

Recall that $\lim_{x \to a} f(x) = L$ means that
As x gets close to a, but with $x \neq a$, the value of $f(x)$ gets close to L

The part about $x \neq a$ becomes important when we try to find limits that result in an expression of $0/0$ when $x = a$ is substituted. When this happens, we try to simplify or use other techniques. The following are examples where simplification is used.

1. $\lim_{x \to -3} \dfrac{x+3}{x^2+4x+3}$
2. $\lim_{x \to -1} \dfrac{\sqrt{x^2+8}-3}{x+1}$

Slide 8 **Sandwich (Squeeze) Theorem (2.2)**

Some problems are too hard to simplify with algebraic manipulations. For these, we can sometimes use the sandwich (or squeeze) theorem:

Theorem: Suppose that

$$g(x) \leq f(x) \leq h(x)$$

for all x in some open interval containing a, except possibly at a itself. Suppose also that

$$\lim_{x \to a} g(x) = L \quad \text{and} \quad \lim_{x \to a} h(x) = L$$

Then

$$\lim_{x \to a} f(x) = L$$

Use the Theorem to show $\lim_{x \to 0} x \sin(\frac{1}{x}) = 0$

Slide 9 **Limits from Graphs (2.2)**

- One can also figure out limits from simple graphs.
- On a graph, an "open circle" means that the function does **not** pass through the corresponding point.
- The idea is to look at what the function "should be" at the given point.

Draw some graphs and try to find the limits.

Slide 10 **One Sided Limits (2.4)**

Sometimes it is useful to find the limit from only one side.

- For the right hand limit only consider $x > a$

$$\text{Notationally:} \quad \lim_{x \to a^+} f(x)$$

- For the left hand limit only consider $x < a$

$$\text{Notationally:} \quad \lim_{x \to a^-} f(x)$$

Draw some graphical examples.

Fact: If

$$\lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L$$

then

$$\lim_{x \to a} f(x) = L$$

Slide 11 **Piecewise Functions (2.4)**

For the function,

$$f(x) = \begin{cases} x, & x < 2 \\ 3 - x, & 2 < x < 3 \\ x^2 - 9, & 3 < x \end{cases}$$

determine if the following limits exist an if so, find their values

1. $\lim_{x \to 2^-} f(x)$
2. $\lim_{x \to 2^+} f(x)$
3. $\lim_{x \to 2} f(x)$
4. $\lim_{x \to 3^-} f(x)$
5. $\lim_{x \to 3^+} f(x)$
6. $\lim_{x \to 3} f(x)$

Slide 12 $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$

We can use the sandwich (squeeze) theorem to prove that

$$\lim_{\theta \to 0^+} \frac{\sin(\theta)}{\theta} = 1$$

Fill in the details of the following outline:
For $\theta > 0$ (and $\theta < \pi/2$)

- A picture shows that $\sin(\theta) < \theta < \tan(\theta)$
- $1 < \frac{\theta}{\sin(\theta)} < \frac{1}{\cos(\theta)}$
- $\cos(\theta) < \frac{\sin(\theta)}{\theta} < 1$
- The result follows from the sandwich theorem.

There are two important general results that follow:

1. $\lim_{h \to 0} \frac{\sin(h)}{h} = 1$
2. $\lim_{h \to 0} \frac{\cos(h) - 1}{h} = 0$