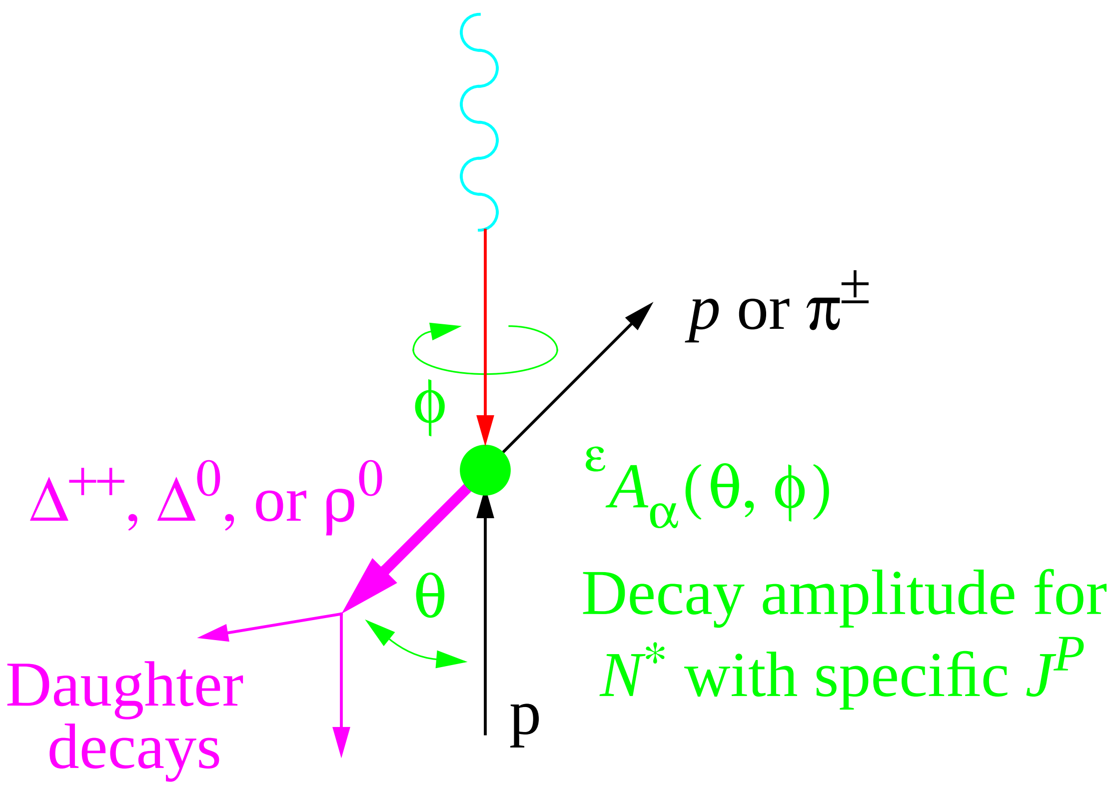
Isobar Model PWA

Model three-body decay as two-body decay followed by subsequent decay of daughter.



Follow decays according to rules of angular momentum, with appropriate rotations and boosts.

Dalitz Plots

Consider a "particle" of mass $M = \sqrt{s}$ which decays to three particles. The decay rate is

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}\overline{|W|}^2 dm_{12}^2 dm_{23}^2$$

where W is the decay matrix element, and m_{AB} is the invariant mass of daughters A,B:

$$m_{AB} = (p_A + p_B)^2$$

(We assume the decaying "particle" has no spin, or that we average over its spin states.)

Therefore, a plot of m_{12}^2 versus m_{23}^2 will be flat if the matrix element is independent of these combinations.

However, if the matrix element depends on "isobars" decaying to these pairs, then this structure will be apparent on the plot.

Comments on $\gamma p \rightarrow p\pi^+\pi^-$

Particular motivation: Missing Baryons

Data is very sparse

CEA, Phys. Rev. 146(1966)994	$\gamma p \rightarrow p\rho$
CEA, Phys. Rev. 146(1966)994	$\gamma p \rightarrow \Delta\pi$
DESY, Phys. Rev. 175(1968)1669	$\gamma p \rightarrow p\pi\pi$...

Complicated Partial Wave Analysis

Dalitz plots
Isobar model

Experiments in Progress at JLab

$\gamma p \rightarrow p\pi^+\pi^-$
$\gamma^* p \rightarrow p\pi^+\pi^-$
$\gamma_{POL} p \rightarrow p\pi^+\pi^-$

Other Topics & References

• Electroproduction and q^2 Evolution

P. Stoler, Phys. Rep. 226(1993)103
S. Capstick, Phys. Rev. D46(1992)2864
V. Frolov, et al., new measurements

• Meson Photo- and Electroproduction

Data is very sparse!

$\gamma p \rightarrow \rho^{\pm}\{n,\Delta\}$

D. P. Barber, et al., Z. Phys. C2(1979)1

$\gamma p \rightarrow \pi\pi\pi\pi p$ (Higher ρ's)

D. P. Barber, et al., Z. Phys. C4(1980)169

$\gamma p \rightarrow \omega^{\pm}\{n,\Delta\}$

D. P. Barber, et al., Z. Phys. C26(1984)343

• Overlapping mesons and baryons

Concentrate on $\gamma p \rightarrow p\pi^{+}\pi^{-}$ ...

Class Exercise and Homework

1) Download data and PAW program from the website, including the code for E_{1+}/M_{1+} ratio formula

2) Use PAW program to fit for free parameters. The code allows for a flat background to be included.

3) Study the fit and draw some conclusions
   - Do you need a background to get a good fit?
   - Can you fit with $E_{1+}=0$? How about $M_{1+}=0$?
   - Can you reconcile this data with the presently accepted value for E_{1+}/M_{1+} ?

Examples

Note: See also R. F. Peierls, Phys. Rev. 118(1960)325

Consider the case of "pure waves"
$\Rightarrow$ Only one J^P contributes

State	Multipole	$I(\theta)$
P33	M_{1+}	$[5-3\cos^2\theta]/2$
	E_{1+}	$[1+\cos^2\theta]/2$
S11	E_{0+}	1
D13	E_{2-}	$[5-3\cos^2\theta]/2$
	M_{2-}	$[1+\cos^2\theta]/2$

Remember: Angular distribution only depends on J for pure waves.

How does this compare to data?

Are there resonances with energy?
$\Rightarrow$ Arndt, et al., Phys. Rev. C(1996)430

First, rearrange some terms...

- Parity Eigenstates

  Helicity amplitudes are of mixed parity

  ⇒ Redefine using parity eigenstates

$$
\begin{aligned}
A_{n+} &= -\frac{1}{\sqrt{2}}(A^{j}_{1/2,\,1/2} + A^{j}_{-1/2,\,1/2}) \\
A_{(n+1)-} &= \frac{1}{\sqrt{2}}(A^{j}_{1/2,\,1/2} - A^{j}_{-1/2,\,1/2}) \\
B_{n+} &= \left[\frac{2}{n(n+2)}\right]^{1/2}(A^{j}_{1/2,\,3/2} + A^{j}_{-1/2,\,3/2}) \\
B_{(n+1)-} &= -\left[\frac{2}{n(n+2)}\right]^{1/2}(A^{j}_{1/2,\,3/2} - A^{j}_{-1/2,\,3/2})
\end{aligned}
$$

with $n=j-1/2$, and $n \geq 1$ for the B's.

- Multipoles

$$
\begin{aligned}
E_{l+} &= [A_{l+} + (l/2)B_{l+}]/(l+1) \\
M_{l+} &= [A_{l+} - ([l+2]/2)B_{l+}]/(l+1) \\
E_{(l+1)-} &= -[A_{(l+1)-} - ([l+2]/2)B_{(l+1)-}]/(l+1) \\
M_{(l+1)-} &= [A_{(l+1)-} + (l/2)B_{(l+1)-}]/(l+1)
\end{aligned}
$$

for $l \geq 1$, and $E_{0+}=A_{0+}$ and $M_{1-}=A_{1-}$.

Partial Wave Analysis

Specific Case of $\gamma p \to \pi N$

"Phenomenolgy for Collisions of Particles with Spin",
M. Jacob and G.C. Wick, Annals of Physics 7 (1959)404

$$A_{\mu,\lambda}(\theta,\phi) = \sum_j A^j_{\mu,\lambda}(2j+1)d^j_{\mu,\lambda}(\theta)e^{i(\lambda-\mu)\phi}$$

for $\mu = \pm 1/2$ and $\lambda = 1/2, 3/2$

Convenient to write $A_{\mu,\lambda}$ as H_i for $i = 1\ldots4$

So, for any $W = \sqrt{s}$, we have $I(\theta) = \sum_i |H_i|^2$

The angular distribution is expanded in a complete, orthonormal basis.

$\Rightarrow$ Do any of the terms **resonate** as a function of energy??

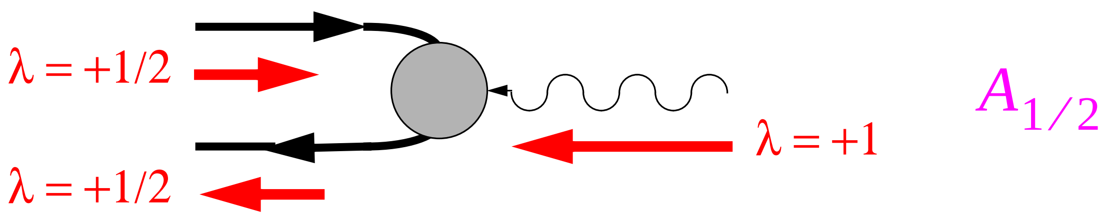
Photoproduction Amplitudes

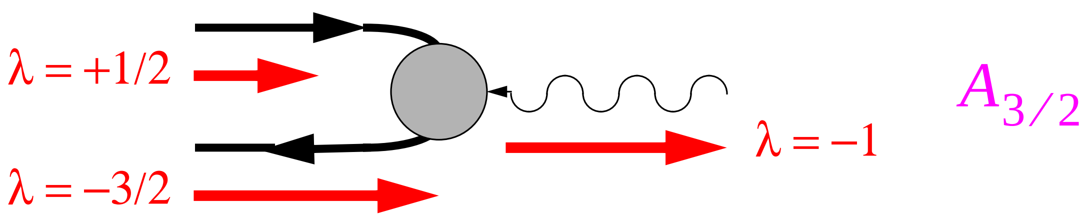
Can follow the same procedure as for πp, but photon spin complicates things.

$\Rightarrow$ Helicity Amplitudes

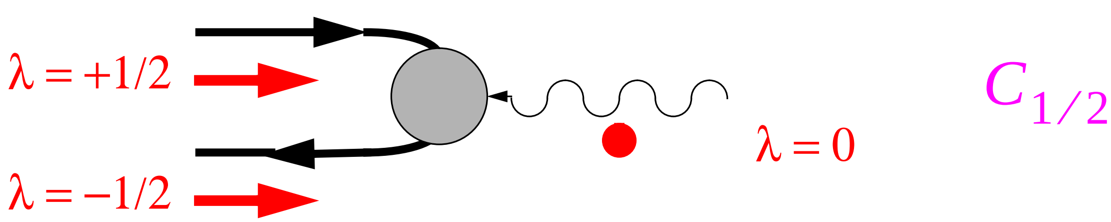
See: R.L. Walker, Phys. Rev. 182 (1969) 1729

Real photons: Two independent amplitudes





Virtual photons: One more amplitude



Now we can predict the angular distribution.

For an unpolarized target and no sensitivity to outgoing nucleon polarization, we have

$$I_J(\theta) = \sum_{m_s} \left\{ \left|\Psi_J^{1/2}\right|^2 + \left|\Psi_J^{-1/2}\right|^2 \right\}$$

This can be evaluated for different *J*.

For *J* = 1/2, find

$$I_{1/2}(\theta) = \text{Constant}$$

For *J* = 3/2 [e.g. the $\Delta(1232)$], find

$$I_{3/2}(\theta) = \text{Constant} \times (1 + 3\cos^2\theta)$$

NOTE: The angular distribution does not depend on the orbital angular momentum of the decay!

$\Rightarrow$ **Look at π^+p at the Δ resonance**

Complete Analysis: Arndt, et al, Phys. Rev. C52(1995)2120

For π+N, specify resonance by {J, L=J ±1/2}

For a given M =±1/2 the amplitude has two orthogonal components. With M =+1/2, find

$$\Psi_J^{1/2} = \langle J\,\tfrac{1}{2}|J-\tfrac{1}{2},0,\tfrac{1}{2},\tfrac{1}{2}\rangle Y^0_{J-1/2}(\theta,\phi)\chi^{1/2}_{1/2}$$

$$+\langle J\,\tfrac{1}{2}|J-\tfrac{1}{2},1,\tfrac{1}{2},-\tfrac{1}{2}\rangle Y^1_{J-1/2}(\theta,\phi)\chi^{-1/2}_{1/2}$$

for the case where L=J -1/2.

Look up the Clebsch-Gordan coefficients in some book on angular momentum, and find

$$\Psi_J^{1/2} = \sqrt{\frac{J+1/2}{2J}}\,Y^0_{J-1/2}(\theta,\phi)\chi^{1/2}_{1/2}$$

$$+\sqrt{\frac{J-1/2}{2J}}\,Y^1_{J-1/2}(\theta,\phi)\chi^{-1/2}_{1/2}$$

and similarly for the other three cases.

Also need to
- determine J^P
- verify resonance phase

⇒ Examine angular distribution

Must incorporate *spin* in the wave function!

With π beam, need $M = \pm 1/2$ for any J
Must still have $s = 1/2$ and $m_s = 1/2$
Choose $L = J \pm 1/2$ depending on Parity

⇒ Tabulate the different possibilities:

L	M	m_l	m_s
J -1/2	+1/2	0	+1/2
		+1	-1/2
	-1/2	-1	+1/2
		0	-1/2
J +1/2	+1/2	0	+1/2
		+1	-1/2
	-1/2	-1	+1/2
		0	-1/2

Introduction

The $\Delta(1232)$ in $\pi^{\pm}p$ Elastic Scattering

Textbook example of a baryon resonance

- Not far above 2π threshold
- Not close to other resonances
- Large cross section

σ_{tot}: Gasiorowicz, Fig.19.3

Isospin I=3/2 ! In terms of $\Psi_{I,\,I_3}$ we write

$$|\pi^+ p\rangle = \Psi_{3/2,\,3/2}$$

$$|\pi^- p\rangle = \sqrt{\frac{1}{3}}\Psi_{3/2,\,-1/2} - \sqrt{\frac{2}{3}}\Psi_{1/2,\,-1/2}$$

$$|\pi^0 n\rangle = \sqrt{\frac{2}{3}}\Psi_{3/2,\,-1/2} + \sqrt{\frac{1}{3}}\Psi_{1/2,\,-1/2}$$

and therefore predict

$$\frac{\sigma_{\pi^+}}{\sigma_{\pi^-}} = \frac{|\langle\pi^+ p|\pi^+ p\rangle|^2}{|\langle\pi^- p|\pi^- p\rangle|^2 + |\langle\pi^0 n|\pi^- p\rangle|^2} = \frac{1}{\frac{1}{9}+\frac{2}{9}} = 3$$

Lecture 3

Baryon Resonances

Reminder: Baryons in the Quark Model

Introduction: The $\Delta(1232)$ in πp Scattering

A clean, isolated resonance

Photoproduction Amplitudes

Formalism and nomenclature

Multipole expansion

Extension to electroproduction

Overlaps with Meson Resonances

The problem with multibody final states

Dalitz plots

s- and *t*-channel pictures

Example and exercise

Fitting $\gamma p \to \pi^+ n$ angular distribution

Note: A very nice (but rather old) reference is
Elementary Particle Physics, S. Gasiorowicz, Wiley (1966)

III: More Sophisticated Treatments

Relativized models

Main point of Godfrey & Isgur
and Capstick & Isgur *but ...*

"It's better to have the right degrees of freedom moving at the wrong speed, than the wrong degrees of freedom moving at the right speed."

Flux tube and $^3P_0$

Promising extensions. See

"Higher Quarkonia",
Barnes, Close, Page, and Swanson
Phys. Rev. D55 (1997)4157

and references therein, especially

[1, 2, 3, 4, 11]

Mixing

Of course, we expect to see states mix with each other if they can. This is accomplished by including "off diagonal" terms in the model. Some particular examples include:

η and η'

Strong mixing of 'singlet' and 'octet'. QCD 'anomalies' are also important

ρ/ω

See Wednesday's homework problem! Good example of isospin violation in the strong interactions.

Baryon Resonances

Many overlapping states. Important to take consider in baryon physics.

$\Rightarrow$ *Tomorrow.*

Spin-Orbit Interaction

We have seen the 'Fisher-Price' model work fairly well, and by adding the 'hyperfine' interaction we can fix most of the mistakes.

Does the spin-orbit interaction matter? Our model is still very crude. It will be hard to include V_{SO} if empirical evidence is weak.

With $\vec{J} = \vec{L} + \vec{S}$ we might expect L=1, S=1 triplets with $V_{SO} \propto$ {-4, -2, 0} for J={0, 1, 2}.

So, look at the data:

$a_0(1450)$	*$f_0(1300)$**	$K_0(1430)$
$a_1(1260)$	$f_1(1285)$	*$K_1(1270)$*/$K_1(1400)$**
$a_2(1320)$	$f_2(1270)$	$K_2(1430)$

*A lot of 'mixing' going on

$\Rightarrow$ Hard to pin down V_{SO}!

Homework Problems

1) Determine the ratios of the masses of the *s*, *c*, and *b* quarks to the mass of the *u* quark, using the *difference* between the masses of the following spin-flip meson pairs:

$\{\pi, \rho\}$

$\{K, K^*\}$

$\{D, D^*\}$

$\{B, B^*\}$

2) Compare these ratios to the ratio of the ϕ, J/ψ, and Υ masses to the mass of the ρ. Comment on the comparison.

3) Assuming a simple model of the form

$$M(q_1\overline{q_2}) = m_1 + m_2 + \frac{k}{m_1 m_2}(\vec{s}_1 \cdot \vec{s}_2)$$

determine a value for k from the mesons in problem 1.

4) Using the value for k you got above, predict the mass difference between the η_c and J/ψ mesons and compare to the measured values. Comment on the comparison.

5) Predict the mass of the η_b from the mass of the Υ. Why will this meson be difficult to discover?

The Spin-Spin ('Hyperfine') Interaction

Quantum Electrodynamics (QED) leads to an interaction between particles with spin due to "one photon exchange":

$$V_{HF} \propto \vec{\mu}_1 \cdot \vec{\mu}_2 \qquad \text{with} \qquad \vec{\mu} = g\frac{e}{m}\vec{s} \propto \frac{\vec{s}}{m}$$

Postulate that a "one gluon exchange" in QCD leads to a potential of the form

$$V_{HF}(q_1\overline{q_2}) = \frac{k}{m_1 m_2}(\vec{s}_1 \cdot \vec{s}_2)$$

$$= \frac{k}{m_1 m_2}\left(-\frac{3}{4}\right) \qquad (S = 0)$$

$$= \frac{k}{m_1 m_2}\left(\frac{1}{4}\right) \qquad (S = 1)$$

Note *at least* qualitative agreement pairs of spin-flip mesons: Splitting $\propto 1/m$

See Godfrey & Isgur, Fig.19

Some Simple Quantum Mechanics

Need eigenvalue of $\vec{j}_1 \cdot \vec{j}_2$ in a $|j_1 j_2 j m\rangle$ basis where $\vec{j} = \vec{j}_1 + \vec{j}_2$. Start with

$$\vec{j}^2 = (\vec{j}_1 + \vec{j}_2)^2 = \vec{j}_1^2 + \vec{j}_2^2 + 2(\vec{j}_1 \cdot \vec{j}_2)$$

and rearrange terms to get

$$(\vec{j}_1 \cdot \vec{j}_2)|j_1 j_2 j m\rangle = \frac{1}{2}[\vec{j}^2 - \vec{j}_1^2 - \vec{j}_2^2]|j_1 j_2 j m\rangle$$

$$= \frac{1}{2}[j(j+1) - j_1(j_1+1) - j_2(j_2+1)]|j_1 j_2 j m\rangle$$

where

$$j = |j_1 - j_2|, |j_1 - j_2| + 1, \ldots, j_1 + j_2$$

Example: Two spin-1/2 particles can couple to give total spin 0 or 1, corresponding to $\vec{s}_1 \cdot \vec{s}_2 = -\frac{3}{4}$ for *S*=0 and $\vec{s}_1 \cdot \vec{s}_2 = \frac{1}{4}$ for *S*=1.

II: The Naive Quark Model

Includes some 'expected' effects

A: Radial potential energy curve

e.g. Godfrey & Isgur, Fig.12
Binding energy differs with radius
Strong perturbation on 3D SHO

B: Spin-Spin and Spin-Orbit Forces

$$V_{HF} \propto (\vec{s}_1 \cdot \vec{s}_2)$$

$$V_{LS} \propto \frac{1}{r}\frac{\partial V}{\partial r}(\vec{L} \cdot \vec{s}_i)$$

C: Mixing

Nearby states with the same quantum numbers may very well mix.

Nearby states with *almost* the same quantum numbers mix if an interaction doesn't respect the quantum symmetry.

This toy model has some great successes...

Hadron	Quarks	Predicted Mass	Measured Mass
ϕ	$s\bar{s}$	1014	1020
D_s^*	$c\bar{s}$	2132	2112
Σ^*	uds	1354	1385
Ω^-	sss	1598	1672
Λ_c^*	udc	2472	~2600
J/ψ	$c\bar{c}$	3250	3095

(Works okay even for heavy quarks, with light quark mesons setting the parameters!)

... and some spectacular failures!

$$M_\pi \neq M_\rho$$

$$M_p \neq M_\Delta$$

η? η'??

Three different ρ's ?

Missing ingredients: V(r), Spin, Mixing, ...

Formulations of the Quark Model

I: The 'Fisher-Price' Toy Model

This is the simplest constituent quark model you can think of:

$$M(q_1\overline{q_2}) = m_1 + m_2 - B$$

$$M(q_1 q_2 q_3) = m_1 + m_2 + m_3 - B$$

Pick two cases to solve for $m_u = m_d$ and B:

$$M(\rho) = 2m_u - B = 770$$

$$M(\Delta) = 3m_u - B = 1232$$

which gives $m_u = 462$ and $B = 154$. Also

$$M(K^*) = m_u + m_s - B = 892$$

$$M(D^*) = m_u + m_c - B = 2010$$

which gives $m_s = 584$ and $m_c = 1702$

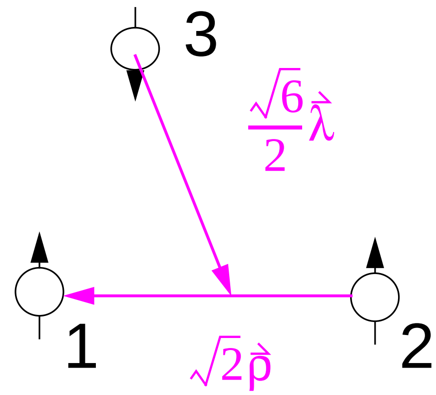
Baryons in the Quark Model

Capstick and Isgur, Phys. Rev. D34(1986)2809

- qqq with orbital angular momentum l

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2)$$

$$\vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$



- Built from 3D Harmonic Oscillator Basis

Excitation energy $E_N = \left(N + \frac{3}{2}\right)h\nu$

Principal quantum number $N = 0, 1, 2, \ldots$

Orbital quantum number and *Parity*:

$N = 0 \qquad P = + \qquad l = 0$

$N = 1 \qquad P = - \qquad l = 1$

$N = 2 \qquad P = + \qquad l = 0, 2$

Number of radial nodes $k = (N - l)/2$

How Many ρ's Are There?

Recall: A 'ρ' has $I^G J^{PC} = 1^+ 1^{--}$

First, expect the "ground state":

$$L = 0 \qquad S = 1 \qquad \Rightarrow \rho(770)$$

In spectroscopic notation, this is 1^3S_1

However I can also get the same quantum numbers in 1^3D_1 , i.e. with

$$L = 2 \qquad S = 1 \qquad \Rightarrow \rho(1700)$$

Finally, it is also possible to "radially excite" the $q\bar{q}$ pair, and get the 2^3S_1

$$L = 0 \qquad S = 1 \qquad (n = 2) \Rightarrow \rho(1450)$$

Many other variations are possible.

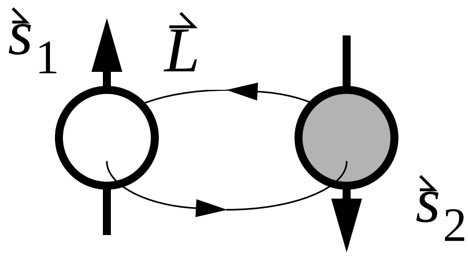
What sets the energy scale??
⇒ Later this lecture ...

Mesons in the Quark Model

Godfrey and Isgur, Phys. Rev. D32(1985)189

- $q\bar{q}$ pairs with orbital angular momentum

$$\vec{S} = \vec{s}_1 + \vec{s}_2$$



- Meson spin $\vec{J} = \vec{L} + \vec{S}$

- Intrinsic Parity $P = (-1)^{L+1}$

- Charge Conjugation $C = (-1)^{L+S}$

  Only possible for neutral mesons!

- G-Parity $G = (-1)^{L+S+I}$

  Only possible for I=0 or 1 (i.e. no K's)

$\Rightarrow$ Build meson *nonets* (i.e. nine orthogonal $q\bar{q}$ states with the same J^{PC}) using different combinations of L and S.

Diquark Model Pediction

Status of the Non-Strange Baryons

State	Rating	$SU(6)\times O(3)$	Parity	State	Rating	$SU(6)\times O(3)$
P_{11} (938)	✡✡✡✡	$(56,0^+)$	+	P_{33} (1232)	✡✡✡✡	$(56,0^+)$
S_{11}(1535)	✡✡✡✡	$(70,1^-)$	-	S_{31}(1620)	✡✡✡✡	$(70,1^-)$
S_{11}(1650)	✡✡✡✡			D_{33} (1700)	✡✡✡✡	
D_{13}(1520)	✡✡✡✡					
D_{13}(1700)	✡✡✡					
D_{15}(1675)	✡✡✡✡					
P_{11}(1440)	✡✡✡✡	$(56,0^+)$	+	P_{31} (1875)	✡✡✡✡	$(56,2^+)$
P_{11}(1710)	✡✡✡	$(70,0^+)$		P_{31} (1835)		$(70,0^+)$
P_{11}(1880)		$(70,2^+)$				
P_{11}(1975)	✡	$(20,1^+)$				
P_{13}(1720)	✡✡✡✡	$(56,2^+)$		P_{33} (1600)	✡✡✡	$(56,0^+)$
P_{13}(1870)	✡	$(70,0^+)$		P_{33} (1980)	✡✡✡	$(56,2^+)$
P_{13}(1910)		$(70,2^+)$		P_{33} (1985)		$(70,2^+)$
P_{13}(1950)		$(70,2^+)$				
P_{13}(2030)		$(20,1^+)$				
F_{15}(1680)	✡✡✡✡	$(56,2^+)$		F_{35} (1905)	✡✡✡✡	$(56,2^+)$
F_{15}(2000)	✡✡	$(70,2^+)$		F_{35} (2000)	✡✡	$(70,2^+)$
F_{15}(1995)		$(70,2^+)$				
F_{17}(1990)	✡✡	$(70,2^+)$		F_{37} (1950)	✡✡✡✡	$(56,2^+)$

Parity

SU(6) x O(3) Representation

Is the Quark Model the Same as QCD?

The answer has to be "NO".

The degrees of freedom in the Quark Model are massive, weakly interacting, spin-$\frac{1}{2}$ objects called "Constituent Quarks".

The degrees of freedom in QCD are light, strongly interacting "Current Quarks" *and* colored, nonlinearly-coupled gluons.

QCD is "right" and the Quark Model is "not".

But the Quark Model works very well, and QCD is nearly impossible to solve.

$\Rightarrow$ Identify the relevant degrees of freedom with models and experiments.

Examples:
- Exotic Mesons
- The Missing Baryons

Lecture 2

The Quark Model

Why bother with it?

Mesons in the Quark Model

How many ρ's do you expect?

Baryons in the Quark Model

Formulations of the Quark Model

The 'Fisher-Price' quark model

The naive quark model

More sophisticated treatments

Class Exercise and Homework

1) Download data and PAW program from the website, including the code for three-resonance formula

2) Use PAW program to fit for free parameters

3) Modify code to study the following:
   - Can you fit the data with two resonances?
   - Can you fit the data with three resonances and fixed phase difference?
   - Fix the $\rho(770)$ mass and width to the PDG values; how much do the other parameters change?
   - Look carefully at the data for $650 \leq \sqrt{s} \leq 900\ \mathrm{MeV}$:

     Does it look like a single Breit-Wigner?

     What is going on here?

     Hint: Fit this portion to two BW's

Fitting Procedure

Fit data (i.e. $|F_\pi|^2$) with the form

$$F_\pi(s) = \sum_{k=1,n} A_k$$

with the amplitude A_k given by

$$A_k = \frac{g_k \exp(i\phi_k)}{s - M_k^2 + iM_k\Gamma_k(s)}$$

and the mass-dependent width $\Gamma_k(s)$ by

$$\Gamma_k(s) \approx \Gamma_{0k}\left(\frac{s - 4m_\pi^2}{M_k^2 - 4m_\pi^2}\right)^{3/2}\left(\frac{M_k}{\sqrt{s}}\right)$$

for the $4n - 1$ parameters $\{M_k, \Gamma_{0k}, g_k, \phi_k\}$

$\Rightarrow$ • **How many resonances n ?**

• **What are their masses, ... ?**

The Pion Form Factor in the Timelike Region

Cross Sections for $e^+e^- \to \pi^+\pi^-$

- Barkov, et al., Nucl. Phys. B256(1985)365 (Novosibirsk)
  Data up to $M \approx 1500$ MeV
  Tabulates all previous data
  Note angular distributions (Fig.3)

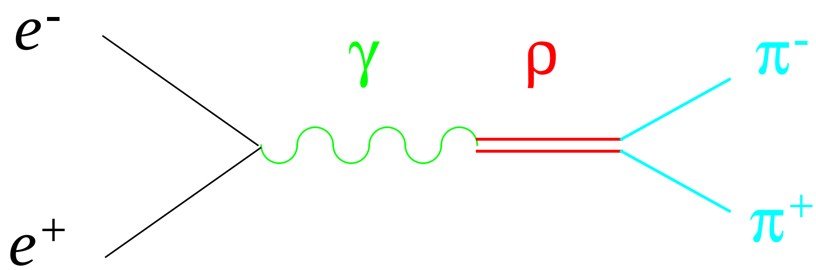
- Bisello, et al., Phys. Lett.B220(1989)321 (Orsay)
  Data up to $M \approx 2400$ MeV

For a more sophisticated fit to the $\rho(770)$ region, see ...

- Bernicha, et al., Phys. Rev. D50(1994)4454

Resonance Analysis: An Example

$$e^+e^- \to \pi^+\pi^-$$



- Timelike Virtual Photons
  "Vector Dominance" $\Rightarrow J^{PC}=1^{--}$
- $G(\pi^+\pi^-)=+1$ and C=-1 so I=1
  We call these "ρ meson resonances"
- Only One Partial Wave to Consider
  $\Rightarrow$ Must have $d\sigma/d\Omega \sim \sin^2\theta$
- Analyze in terms of "Pion Form Factor":

$$\sigma(e^+e^- \to \pi^+\pi^-) = \frac{\pi}{3}\frac{\alpha^2}{s}\beta_\pi^{\,3}|F_\pi|^2$$

$$= \sigma(e^+e^- \to \mu^+\mu^-)\frac{\beta_\pi^{\,3}}{4}|F_\pi|^2$$

The Relativistic Breit-Wigner Formula

See: *An Intro to Quantum Field Theory*, Peskin & Schroeder (1995)
Elementary Particle Physics, S. Gasiorowicz (1966)
J.D. Jackson, Nuovo Cimento 34(1964)1644

The amplitude for resonance production is

$$A = \frac{g_A \exp(i\phi_A)}{s - M^2 + iM\Gamma(s)}$$

where $s = p^{\mu}p_{\mu} = p^2$ is the square of the four-momentum of the resonance ‘particle’.

Decay rate depends on mass, so $\Gamma = \Gamma(s)$,

$$\Gamma(\omega) \approx \Gamma_0 \left(\frac{q}{q_0}\right)^{2l+1} \left[\frac{\rho(\omega)}{\rho(\omega_0)}\right]$$

where q is the (CM) three momentum in the decay, l is the orbital angular momentum of the decay, and $\omega = \sqrt{s}$. Also, $q = q_0$ and $\omega = \omega_0$ when $s = M^2$, and $\rho(\omega)$ is a slowly varying function that is determined either empirically, or ‘theoretically’ for a specific resonance and decay.

Connection to Particle Physics

A slightly 'handwaving' approach

- Interpret "resonance" as an unstable particle of mass *M* and four-momentum p^μ
- Want relativistically covariant formula
- Consider the term $p^2 - M^2 + iM\Gamma$ with

$$p^2 - M^2 = p^{0\,2} - (\vec{p}^{\,2} + M^2)$$
$$= (p^0 + E_{\vec{p}})(p^0 - E_{\vec{p}})$$
$$\approx (2E_0)(E - E_0)$$

for a resonance in *energy* near E_0. Then

$$p^2 - M^2 + iM\Gamma \approx 2E_0\left[E - E_0 + i\frac{(M/E_0)\Gamma}{2}\right]$$

$\Rightarrow$ *This reduces to the nonrelativistic form near the resonance and with the proper 'time-dilated' width.*

Quantum Mechanical Resonance

"Unbound" states of a potential have a finite lifetime τ and therefore a finite "width" Γ:

$$\Gamma = 1/\tau$$

These states decay away according to

$$|\Psi(t)|^2 = |\Psi(0)|^2 e^{-t/\tau} = |\Psi(0)|^2 e^{-\Gamma t}$$

So, the state with eigenvalue $E_R = \omega_R$ is

$$\Psi(t) = \Psi(0) e^{-i\omega_R t} e^{-(\Gamma/2)t}$$

$$= \Psi(0) e^{-t(iE_R + \Gamma/2)}$$

To study the process in terms of *energy* (and not *time*), take the Fourier Transform:

$$\chi(E) = \int_0^\infty \Psi(t) e^{i\omega t} dt$$

$$= \Psi(0) \int_0^\infty e^{[i(E - E_R) - \Gamma/2]t} dt$$

$$= \frac{i\Psi(0)}{(E - E_R) + i(\Gamma/2)}$$

Resonance in Electrical Oscillations

An analysis based on "impedance"

$$\frac{V_{\text{OUT}}}{V_{\text{IN}}} = \frac{Z_{LC}}{Z_R + Z_{LC}}$$

$$= \frac{\left(\frac{1}{Z_L} + \frac{1}{Z_C}\right)^{-1}}{Z_R + \left(\frac{1}{Z_L} + \frac{1}{Z_C}\right)^{-1}}$$

$$= \frac{-i(\omega/\tau)}{\omega^2 - \omega_0^2 - i(\omega/\tau)}$$

$$= ge^{i\phi}$$

where $Z_R = R$, $Z_L = i\omega L$, and $Z_C = \frac{1}{i\omega C}$,

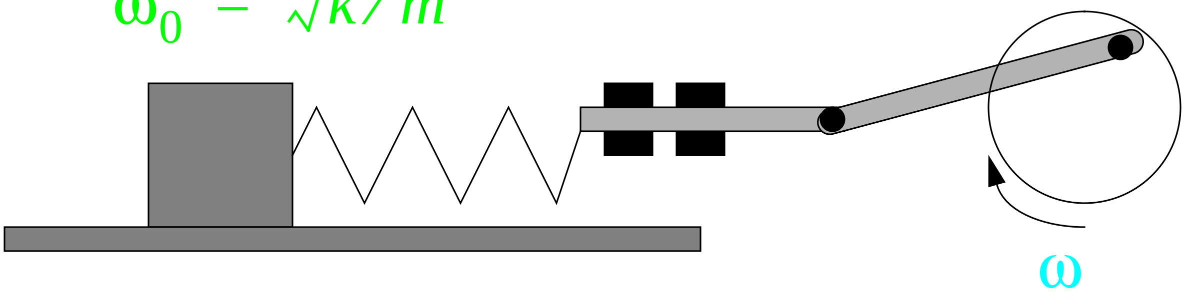
and we write $\omega_0^2 = 1/(LC)$ and $\tau = RC$

$\Rightarrow$ How do the *amplitude* g
and the *phase* ϕ
depend on *frequency* ω ?

Classical Resonance

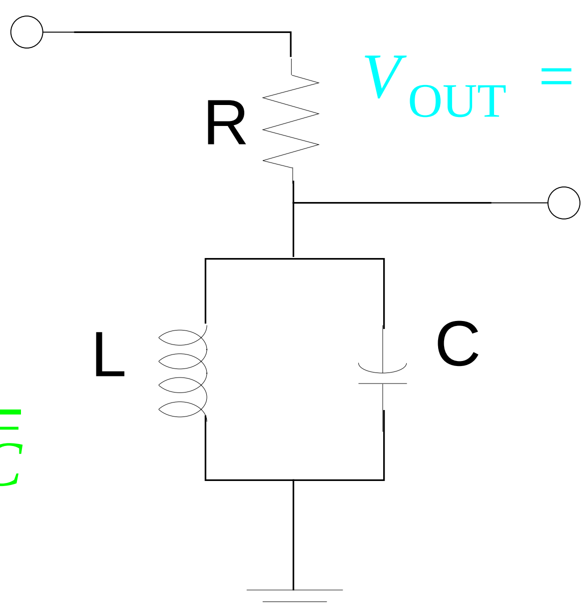
Mechanical Oscillations

$$\omega_0 = \sqrt{k/m}$$



Electrical Oscillations

$$V_{\mathrm{IN}} = V_0 e^{i\omega t}$$



$$V_{\mathrm{OUT}} = g V_0 e^{i(\omega t + \phi)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

"Bump Hunting" Is NOT Enough!

Many overlapping states.

Need to identify "resonances" with separate quantum numbers.

"Partial Wave Analysis"

We'll analyze some simple examples,

but first...

... we'll study the idea of "resonance".

Examples from Data

γp and γd Total Cross Section

Particle Data Group, Phys Rev D54(1996)I
M. MacCormick, et al, Phys Rev C53(1996)41

$\gamma *$ Total Cross Section

P. Stoler, Phys Rev D44(1991)73

$\gamma p \rightarrow n\pi^{+}$ Total Cross Section

D. Menze, et al, Data Compilation(1977)
Cambridge BC, Physical Review 155(1967)1477

$\gamma p \rightarrow p\pi^{+}\pi^{-}$ Total Cross Section

Cambridge BC, Physical Review 146(1966)994
Cambridge BC, Physical Review 155(1967)1477
Cambridge BC, Physical Review 163(1967)1510
ABBHHM, Physical Review 175(1968)1669

$e^{+}e^{-} \rightarrow \pi^{+}\pi^{-}$ Total Cross Section

L.M. Barkov, et al, Nuclear Physics B256(1985)365
D. Bisello, et al, Physics Letters B220(1989)321

Real and Virtual Photons

Notation and Language

Real Photons

- "Photoproduction": $q^2 = 0$
- Meson resonances are produced by "exchange"
- Baryon resonances are "formed"

Spacelike Virtual Photons

- "Electroproduction": $q^2 < 0$
- Otherwise analgous to photoproduction

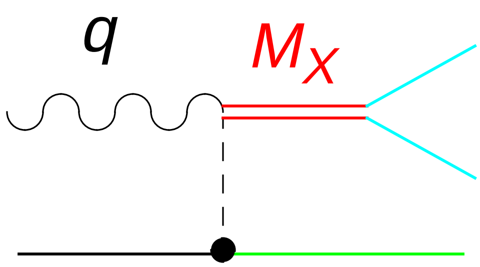
Timelike Virtual Photons

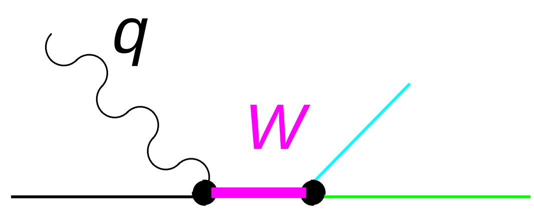
- "e^+e^- Annihilation": $q^2 > 0$
- Only meson resonances

Real and Virtual Photons

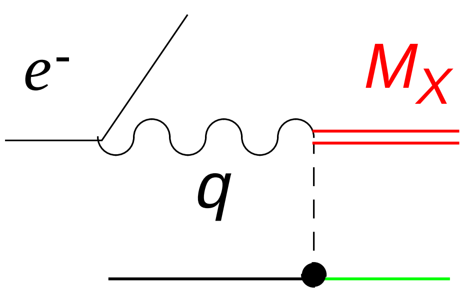
Diagrams of Resonance Formation

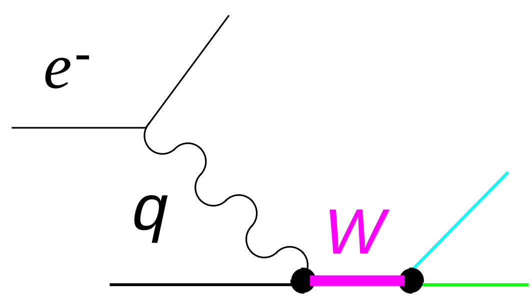
Real Photons



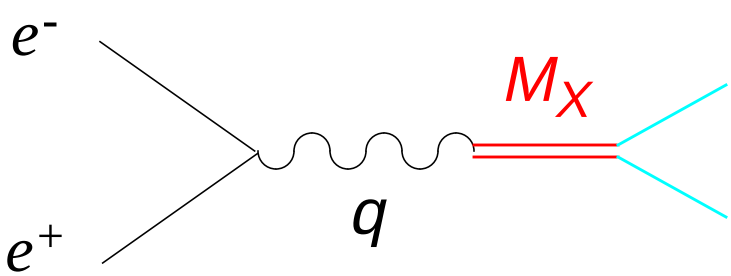


Spacelike Virtual Photons





Timelike Virtual Photons



Electro- and Photoproduction of Meson and Baryon Resonances

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Wednesday May 27 (Two Hours)

- Real and Virtual Photoproduction
- Resonance production
- Analysis of $e^+e^- \to \pi^+\pi^-$

Thursday May 28 (One Hour)

- The Quark Model
- Excited States of Hadrons

Friday May 29 (Two Hours)

- Baryon Resonances
- Photoproduction Amplitudes
- Multibody final states