

Errata: Typographical Errors, Mistakes, and Comments  
Modern Quantum Mechanics, 2nd Edition  
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May 17, 2012

**Page 2.** *Figure 1.1.* The figure has the north pole on top and the south pole below, reversed from the analogous figure in the Revised Edition. The text on page 3 needs to be modified.

**Page 5.** *Figure 1.3.* The dotted lines labeled “ $S_x$ – beam” and “ $S_z$ – beam” should be solid lines instead. The dotted line labeled “No  $S_z$ – comp.” in the uppermost drawing can stay.

**Page 34.** Inside the proof for Lemma 1.1, the sentence between (1.4.56) and (1.4.57) should read instead “This inequality must hold when  $\lambda$  is set equal to  $-\langle\beta|\alpha\rangle/\langle\beta|\beta\rangle$ .”

**Page 35.** The inequality (1.4.59) should close on the left side with an angle bracket, i.e.  $\rangle$ , rather than a right parenthesis.

**Page 58.** *Problems 1.2 and 1.3.* The Pauli matrices  $\sigma$  are not defined in Chapter One. See (3.2.32). Perhaps these problems should be moved to Chapter Three.

**Page 60.** *Problem 1.11.* The hint would appear to be misplaced, referring essentially to the solution of Problem 1.9, the connection is peculiar. Following that hint can help you solve the problem, but would nonetheless be confusing. I think the hint should just be scrapped.

**Page 60.** *Problem 1.12.* This problem is a special case of Problem 1.9, and with different notation for the angles. It would make sense to combine it with Problem 1.9.

**Page 62.** *Problem 1.19.* The first term on the right hand side of the equation should be  $\langle S_x^2 \rangle$ , not  $\langle S_2^x \rangle$ .

**Page 63.** *Problem 1.24.* See (3.2.32) for a definition of  $\sigma_x \equiv \sigma_1$ .

**Page 93.** In (2.3.31), the second factor on the right should be  $(1/\sqrt{2}x_0)^2$ , not  $(1/\sqrt{2x_0})^2$ .

**Page 108.** *Equation (2.5.29).* The integral is missing a “ $dx$ ”.

**Page 114.** *Equation (2.5.49).* Delete the minus sign between the right bracket and  $\cos$ .

**Page 118.** *Equation (2.6.15).* The first  $p$  inside square bracket should be  $p'$

**Page 119.** *Equation (2.6.18).* There is a missing left parenthesis before  $x''^2$ .

**Page 121.** In the first full sentence of the paragraph following (2.6.26), the reference should be to Section 2.2, not Section 2.1.

**Pages 135–137.** The kinematical momentum  $\mathbf{\Pi}$  is a vector and should be in bold face.

**Page 145.** *Equation (2.7.76).* The first term is incorrect; the divisor  $r \sin \theta$  should be in front of the square brackets. Also, the unit vector  $\hat{\boldsymbol{\theta}}$  should be in bold font.

**Page 148.** A problem should be added that treats the linear potential (2.5.30) through Schrödinger’s equation in momentum space.

**Page 149.** *Problem 2.3.* Part (a) should read  $S_x = \hbar/2$ , i.e. upper case  $S$ .

**Page 150.** *Problem 2.11.* This problem is rather open ended, atypical for most of the problems in the book. Perhaps it should be revised. Most of the problem is in fact covered on pages 94 to 96.

**Page 151.** *Problem 2.12.* Not enough information is given in the problem statement. The state  $|0\rangle$  is one for which  $\langle x \rangle = 0 = \langle p \rangle$ .

**Page 151.** *Problem 2.14.* Evaluating  $\langle m|\{x,p\}|n\rangle$  is rather tedious and, to my knowledge, not particularly enlightening or useful. It could be removed. Also, it is worth stating the virial theorem here for convenience. That is

$$\left\langle \frac{\mathbf{p}^2}{m} \right\rangle = \langle \mathbf{x} \cdot \nabla V \rangle \quad \text{in 3D, or} \quad \left\langle \frac{p^2}{m} \right\rangle = \left\langle x \frac{dV}{dx} \right\rangle \quad \text{in 1D}$$

**Page 152.** *Problem 2.19.* There is a lot to say about this problem.

- The problem is really about quantum optics. See the end Chapter Seven of the text-book. (This section should be expanded in future editions.) Also see R. J. Glauber, Phys.Rev.84(1951)395 and his Nobel lecture in Rev.Mod.Phys.78(2006)1267; Gottfried 1966, Sec.31; Merzbacher 1998, Sec.10.7; and Gottfried 2003, Sec.4.2.
- It should be stated that  $P_n(\mu) = e^{-\mu}\mu^n/n!$  is the Poisson distribution of integers  $n$  with a mean  $\mu$ . Note that the “most probable” value of  $n$  is an integer.

- Part (d) includes a reference to Gottfried 1966, 262–264. This is for a theorem (which Gottfried 2003 calls the Baker-Campbell-Hausdorff theorem) needed to solve it, namely that  $e^{A+B} = e^A e^B e^{-[A,B]/2}$  for any two operators  $A$  and  $B$  which commute with  $[A, B]$ . See also Problem 2.13 in Gottfried 2003, and Equation 39 in Glauber 1951.

**Page 152.** *Problem 2.20.* This problem borrows definitions from Chapter 3, and in fact provides some of the background for Section 3.9, but a proper definition of  $\mathbf{J}^2$  is not provided up to this point. Given the context of this problem, it is probably best to explicitly add the definition of  $\mathbf{J}^2$  as given in the first line of (3.9.12), namely

$$\mathbf{J}^2 \equiv J_z^2 + \left(\frac{1}{2}\right)(J_+ J_- + J_- J_+)$$

**Page 153.** *Problem 2.23.* The problem does not appear to be well-posed. From the existing solutions manual, I gather that “known to be exactly at  $x = L/2$  with certainty” means that the wave function is proportional to  $\delta(x - L/2)$ , but this does not appear to be normalizable. I would suggest dropping this problem in future editions.

**Page 153.** *Problem 2.25.* I think there is nice physics in this problem, but neither the presentation nor the solution manual are enlightening. First, it should be combined with problem 24, and the notation made the same. The (negative) energy eigenvalue for  $t < 0$  is unphysical for  $t > 0$ , so the answer needs to be expanded in terms of positive energy states. I want to study this more before deciding how to rewrite the problem, and the solution.

**Page 154.** *Problem 2.27.* The second sentence “Your answer... in two dimensions” can be deleted. This problem is held over from the Revised Edition, but the Second Edition explicitly covers the density of states in three dimensions in this chapter.

**Pages 154, 155.** The old solutions manual gives answers for Problems 2.31 through 2.35 for which I am not entirely clear. I suspect that the next revision will see these problems, and also Section 2.6, significantly reworked.

**Page 156.** *Problem 2.40.* The label says  $\rho = h/\lambda$ , but it should be (momentum)  $p = h/\lambda$ . Also, in the formula one is asked to derive,  $\Delta B$  is not defined and should simply be  $B$ , and  $\bar{\lambda}$  should be “ $\lambda$ -bar”, i.e.  $\lambda/2\pi$ .

**Page 191.** *Equation (3.5.4b).* The eigenvalue should be  $b$ , not  $a$ .

**Pages 208-209.** One of my favorite lessons from Dirk Walecka is the physical reasoning that allows one to eliminate the solutions  $u(r) \propto r^{-l}$ . In my haste to get to the physics, though, I neglected to include the spherical harmonic factors in (3.7.13). Of course, integrating over the sphere removes the angular dependence, and the result is the same.

**Page 227.** There are some problems here. First, two errors were made in transcribing to

the Second Edition from the Revised Edition. Just above (3.8.54), the text should refer to (3.8.49) instead of (3.7.49). Also, the second factor in the square root in the bottom part of (3.8.54) should be  $(l - m + 1/2)$  instead of  $(l - m - 1/2)$ . These changes, then, correctly lead to (3.8.56). I am unable, however, to derive (3.8.54) from (3.8.49), as  $m$  seems to be confused with  $m + 1$ . I will need to look into this subsection further.

**Page 230.** *Equation (3.8.68).* On the right hand side, the symbols  $\otimes$  should be  $\oplus$ .

**Page 245.** The first sentence of the last paragraph had some errant words added at the proofing stage. It should read “It might be thought that A and B can communicate if one of them suddenly changes the orientation of his or her analyzing apparatus.”

**Page 251.** The end of the paragraph at the bottom of the page would be clearer if it were written as “ $k_{1,2} \rightarrow j_{1,2}$  and  $q_{1,2} \rightarrow m_{1,2}$ .”

**Page 255.** I should add a problem or two to this chapter having to do with normalized eigenfunctions of the isotropic 3D harmonic oscillator.

**Page 256.** *Problem 3.7.* I don’t understand this problem. Either that, or it is trivial. The original solution manual doesn’t clear things up for me, either.

**Page 257.** *Problem 3.15.* I see no good reason to write the equation in (b) as if the  $J_-$  operators acts on a wave function.

**Page 258.** *Problem 3.19.* Spherical harmonics are written as  $Y_{l,m}(\theta, \phi)$  instead of  $Y_l^m(\theta, \phi)$  as they are in the body of the text. I need to go through all instances and make sure that the nomenclature is consistent.

**Page 259.** *Problem 3.22.* The problem is incomplete; there is a step missing. Part (e) should be included which defines the “Associated” Laguerre polynomials as

$$L_n^k(x) = (-1)^k \frac{d^k}{dx^k} [L_{n+k}(x)]$$

and then asks to show that they satisfy the differential equation

$$xL_n^{k''}(x) + (k + 1 - x)L_n^{k'}(x) + nL_n^k(x) = 0$$

which in fact can be shown to be Kummer’s equation. That is, the hydrogen atom radial wave functions are expressed in terms of *associated* Laguerre polynomials, as opposed to Laguerre polynomials.

**Page 259.** *Problem 3.24.* This is a good problem, but I would reword it as follows: Find all nine states  $|j, m\rangle$  for  $j = 2, 1$ , and  $0$  formed by adding  $j_1 = 1$  and  $j_2 = 1$ . Use a simplified notation, where  $|j, m\rangle$  is explicit and  $\pm, 0$  stand for  $m_{1,2} = \pm 1, 0$ , respectively, for example

$$|1, 1\rangle = \frac{1}{\sqrt{2}}|+0\rangle - \frac{1}{\sqrt{2}}|0+\rangle$$

You may also want to make use of the ladder operators  $J_{\pm}$ , or recursion relations, as well as orthonormality. Check your answers by finding a table of Clebsch-Gordan coefficients for comparison. (*I should probably include such a table in future editions of the book.*)

**Page 260.** *Problem 3.27.* I think this problem should be deleted. I don't know what is meant by  $|jmn\rangle$ , and the solution manual's answer, although simple, is unenlightening.

**Page 261.** *Problem 3.32(a).* This is better written as "Write  $xy$ ,  $xz$ , and  $(x^2 - y^2)$  in terms of components of  $a \dots$ "

**Page 265.** This method of solving the hydrogen atom was first worked out by Pauli, in *Zeitschrift Phys.*, **33**(1925)879. An English translation "On the hydrogen spectrum from the standpoint of the new quantum mechanics", is published in *Sources of Quantum Mechanics*, B. L. Van der Waerden, Dover (1967). Many thanks to Djordje Minic for pointing me to this reference.

**Page 265.** Regarding the classical treatment of the Runge-Lenz vector, I should have referred the reader to Goldstein, Poole, and Safko (2002) section 3.9.

**Page 290.** *Figure 4.12.* In (a) the arrow next to "Momentum before reversal" should in fact be attached to "Momentum after reversal." That is, both arrows should come from "after" while "Momentum before reversal" just labels the rightward arrow at  $t = 0$ . Indeed, the labeling in the corresponding Figure 4.11 in the Revised Edition is a bit ambiguous.

**Page 297.** The statement that (4.4.75) "is evident from the properties of angular momentum..." is too cavalier. The phase factor  $(-1)^{2j}$  is not obvious for all  $j$ . However, it can be shown to equal  $d_{mm}^{(j)}(2\pi)$  from the definition (3.5.51) and evaluated with (3.9.33). This could be turned into a problem in Chapter Three or Four in the next edition.

**Page 297.** The statement near the bottom asks a question regarding how one might "visualize  $|j, m\rangle$  as being built up of 'primitive' spin  $\frac{1}{2}$  objects" but the intent of this statement is not clear to me. I would strike the statement, and focus instead on justifying (4.4.79) through compatibility with rotation formalism, as we do in Problem 10.

**Page 298.** In the top paragraph, the reference to (4.4.70) should be (4.4.72). Also, the reference to Frauenfelder and Henley (1974) should be changed to Henley and Garcia (2007). (The time reversal formalism in the later edition appears to be essentially unchanged.)

**Page 300.** *Problem 4.1.* Just to be clear, a "box of side  $L$ " means a three-dimensional cube.

**Page 300.** *Problem 4.3.* The state in question should be written as  $|\Psi\rangle$ .

**Page 301.** *Problem 4.9.* The time reversed state should be written  $\Theta|\alpha\rangle$ .

**Page 301.** *Problem 4.10.* Again, the time reversal operator should be written  $\Theta|\alpha\rangle$ , but this problem has too many idiosyncrasies and should be rewritten. (Problems 8 and 9 also have idiosyncrasies, which the interested reader should identify.) For one thing, we are asked to “prove” a phase convention, which is in fact not used by everyone. Furthermore, the problem does not dovetail neatly into the reference to it at the bottom of page 297. Following is a better restatement of Problem 4.10:

(a) Use (4.4.53) to show that  $\Theta|jm\rangle$  equals  $|j, -m\rangle$  up to some phase that includes the factor  $(-1)^m$ . That is, show that  $\Theta|jm\rangle = e^{i\delta}(-1)^m|j, -m\rangle$ , where  $\delta$  is independent of  $m$ .

(b) Using the same phase convention, find the time-reversed state corresponding to  $\mathcal{D}(R)|jm\rangle$ . Consider using the infinitesimal form  $\mathcal{D}(\hat{\mathbf{n}}, d\phi)$  and then generalize to finite rotations.

(c) From these results, prove that, independent of  $\delta$ , one finds

$$\mathcal{D}_{m'm}^{(j)*}(R) = (-1)^{m-m'}\mathcal{D}_{-m',-m}^{(j)}(R)$$

(d) Conclude that we are free to choose  $\delta = 0$ , and  $\Theta|jm\rangle = (-1)^m|j, -m\rangle = i^{2m}|j, -m\rangle$ .

**Chapter Five.** I remain uncomfortable with the makeup of material in this chapter. Calling it “Approximation Methods” implies that approximations are not introduced elsewhere in the book, but that is not the case. Also, Section 5.5, for example, is not about approximations, and should really be moved to Chapter Two. The most interesting material in this chapter has to do with real-world applications of quantum mechanics. Food for thought.

**Pages 328–330.** Why write  $|\mathbf{B}|$  instead of just  $B$ , as defined in the line following (5.3.33)?

**Page 339.** Equation (5.5.15) is missing the “=” between  $c_n(t)$  and the summation sign.

**Page 342.** The equation for  $H_0$  in (5.5.28) is missing a “–” sign.

**Page 347.** The numerator in (5.6.11) should be the *magnitude* of the matrix element.

**Page 352.** The reference to D.J. Richardson et al. in the last paragraph should be *Phys. Rev. Lett.* (There is an extra “ $t$ ” as printed.) as printed.)

**Page 354.** In (5.6.44) replace  $\mathbf{V}_{\mathbf{R}}$  with  $\nabla_{\mathbf{R}}$  in the second line of the equation.

**Sections 5.7 & 5.8.** The incorporation of the density of states into transition probabilities and cross sections is not done smoothly. This should be rewritten, including better connection to the material in Section 2.5.

**Pages 376–385.** In the problem statements, it is not necessary to continually quote things like the harmonic oscillator matrix element  $\langle n'|x|n\rangle$  and hydrogenic atom wave functions.

**Page 376.** *Problem 5.1.* Remove the last sentence “You may assume...” and the equation that follows. It is misplaced in the problem statement, uses a notation not from this book

for state vectors, and there is no point in the first place as this is all covered in Section 2.3. Also, the problem statement would be neater if the  $\lambda$  was removed from the left side of the equation that defines the perturbation.

**Page 376.** *Problem 5.2.* For notational consistency, the problem should ask for terms up to order  $\lambda^2$ , not  $g^2$ .

**Page 376.** *Problem 5.4.* As discussed above for Problem 5.1, remove the statement in square brackets at the end of part (c).

**Page 377.** *Problem 5.6.* This is a nice problem, taken directly from Problem 17.7 in Merzbacher (1970), maintaining an open ended style. It should be rewritten in the more explicit manner of this book. For example,  $\omega_x = \omega_y \equiv \omega$  and  $\omega_z = (1 + \epsilon)\omega$  with  $\epsilon \ll 1$ . (I would also put the magnetic field in the  $z$ -direction since it simplifies the angular momentum algebra.) Reference should also be made to Section 3.7, Problem 3.21, and also Problem 2.14.

**Page 378.** *Problem 5.12.* There is a lot of physics here, relegated to a brief comment on page 319 after “A challenge for the experts.” This kind of degeneracy is lifted only by distant states. In future editions, I would like to expand this discussion, but for now, see Problem 1, page 397 of Gottfried (1966), and Section 3.7 (page 155) of Gottfried and Yan (2003).

**Page 379.** *Problem 5.16.* I don’t know why this problem is in Chapter 5. Perhaps move it to Chapter 3. I would also like to find a reference for the comment about the  $q\bar{q}$  potential.

**Page 379.** *Problem 5.17.* Part (a) should also ask for a comparison to the exact solution, as it does in Merzbacher. Furthermore, parts (a) and (b) should be separate problems; part (b) isn’t even connected to this chapter. In fact, it is somewhat a rehash of Problem 3.32.

**Page 379.** *Problem 5.18.* It seems unnecessary to explicitly include the ground state hydrogen wave function or the given definite integral in the problem statement.

**Page 380.** *Problem 5.19.* This should be part (b) of Problem 5.18.

**Page 380.** *Problem 5.20.* It is unnecessary to include the integral.

**Page 380.** *Problem 5.21.* Some space is needed between “for” and  $|x|$  in the first equation. Also, we don’t need to include the numerical values given at the end of the problem.

**Page 380.** *Problem 5.22.* The interpretation is cleaner if the sign is reversed on  $V(t)$  so the “forcing function” is  $F(t) = +F_0 \cos \omega t$ . A comparison to classical physics is also warranted.

**Page 382.** *Problem 5.28.* There is nothing onerous about the radial integrals. They can be done out. Also, it is worth the work to consider the limit  $\tau \rightarrow \infty$ .

**Page 383.** *Problem 5.31.* The problem should more properly refer to (5.7.17) and then an

analogy to the “second term” in (5.7.36). The point is to fill in the blanks leading up to Section 5.9. This overall discussion could be cleaner.

**Page 383.** *Problem 5.32.* I don’t understand why part (c) is not just another question inside part (a).

**Page 384.** *Problem 5.34.* The question of photon polarization is not clearly answered, so far as I can tell, in the context of the semi-classical treatment of radiation in this and other books on nonrelativistic quantum mechanics. The best discussion I’ve found is §45 in Schiff (1968). I would like to revisit this problem in the next edition.

**Page 384.** *Problem 5.37.* The wording is clumsy. The neutron is in a quantum state such that its spin is aligned with a magnetic field “fixed at an angle  $\theta$  with respect to the  $\mathbf{z}$ -axis, but rotating slowly in the  $\phi$ -direction.” Ignore the beginning of the next sentence which talks about “the tip of the magnetic field.”

**Page 385.** *Problem 5.39.* This problem should be moved to Chapter Two, probably as part of Problem 2.27.

**Page 385.** *Problem 5.41.* There is a lot of physics in this problem that is barely covered in the text, including normalization of the electromagnetic field. The problem should be expanded, or material should be added to the textbook.

**Page 403.** The last paragraph just before the new subsection should start with “At high energies ( $ka \gg 1$ )...”

**Page 414.** *Equation (6.4.50).* The numerator of the first term should be  $e^{2i\delta_l} e^{ikr}$ .

**Page 424.** *Equation (6.6.5).* A student points out to me that since  $j_0(kr) = \sin(kr)/kr$  and  $n_0(jr) = -\cos(kr)/kr$ , this is a strict equality, not “approximate”. This is from the previous edition, however, and the point may have been that we are continuing an approximate approach, namely low energy scattering. So, is this or is this not a typographical error? De gustibus non est disputandum.

**Page 442.** *Problem 6.1.* The wording “ $E \rightarrow E + i\varepsilon$  prescription” is held over from the Revised Edition, and is irrelevant here. You only need the “(+)” solution. For the last part, refer to Problem 2.24. This alludes to some nice physics, which might be worth exploring in the next edition.

**Page 443.** *Problem 6.6.* This problem should be moved to Chapter 3.

**Page 445.** *Problem 6.13.* The solution to this problem is interestingly connected to the discussion of  $SO(4)$  symmetry and the hydrogen atom, starting on page 265. Perhaps this problem should be reworked and moved to Chapter Four. It is not clear to me what it has

to do with scattering.

**Page 460.** In the sentence at the bottom, just above (7.5.10), the word is “multiparticle” not “mutliparticle.”

**Page 460.** Equation (7.5.9) should have “ $\dots$ ” in the numerator, not “.”.

**Page 463.** *Table 7.1.* The commutator in the second line under **Bosons** should instead read  $[a_i, a_j] = 0$ .

**Page 483.** *Caption on Figure 7.10.* The citation is not quite correct. The journal should read “*Jour. Opt. Soc. Am.B* 4(1987)1465.”

**Page 484.** *Problem 7.8.* The electronic configuration should be written as “ $(1s)^2 \dots$ ” instead of “ $(ls)^2 \dots$ ”. I would also have preferred the first sentence hyphenate “spin- $\frac{3}{2}$ ”.

**Page 491.** In the definition of  $D_\mu$  just following (8.1.14), where it is first used, I think the sign should be reversed, i.e.  $D_\mu \equiv \partial_\mu - ieA_\mu$ . The Revised Edition used the unfortunate nomenclature that  $e$  was the electron charge, that is, a *negative* number. Most texts with which I am used to, define  $e = +4.80325 \times 10^{-10}$  esu, so that the charge on the electron is  $-e$ . I tried to stick with the definition that  $e$  is negative, but I likely missed some places.

**Page 495.** Fourth line from the bottom, it is the “identity”, not “identify”, matrix.

**Pages 515-516.** Problems 8.1 through 8.7 are mis-numbered as 8.11 through 8.17.

**Page 515.** *Problem 8.1(a) (misnumbered as 8.11(a)).* The exponent on the value for the proton mass should be  $-27$ , not  $27$ .

**Page 516.** *Problem 8.7 (misnumbered as 8.17).* There are several typos. Delete the last sentence “Work with the upper component of the wave function.” from the stem. In part **(a)** the wave function should be written as  $e^{-iEt}[u_l(r)/r]Y_{lm}(\theta, \phi)$ , not  $u_l(kr)$ . Also,  $\rho = \gamma r$ , not  $\rho = kr$ . In part **(c)** delete the first occurrence of the word “will”; that is, the end of the sentence should read “. . . terminates, the function will have an incorrect asymptotic form.”

**Page 519.** I am much more pleased with a writeup I made subsequently, discussing units in electromagnetism, than I am with this appendix. See <http://www.rpi.edu/dept/phys/Courses/PHYS4210/S10/NotesOnUnits.pdf>

**Page 531.** This entry in the Appendix must be modified to connect with (3.7.58) and Problem 3.22. For now, beware a misprint and a caveat. The misprint is in the definition of the scaled radius, which should read  $\rho = 2Zr/na_0$ . The caveat is that there are different conventions for defining and normalizing the associated Laguerre polynomial. The one used here is directly from Schiff (1968), but differs from, for example, that used in MATHEMATICA. Equations (B.6.7) are correct as written, and can be used to test one’s choice of convention.

**Page 533.** In the last sentence, the expressions should read  $j_1 \geq j_2$ .

**Page 535.** Include an appendix with some Clebsch-Gordan coefficients, for example <http://pdg.lbl.gov/2011/reviews/rpp2011-rev-clebsch-gordan-coefs.pdf>

**Page 536.** *Lectures on Quantum Mechanics*, by Gordon Baym is now published in paperback by Westview Press (1974). The ISBN is 9780805306675.