PHYS6530 *Quantum Mechanics III*

Fall 2005 Problem Set #7 (Final Problem Set)

Due in the Physics Department Office by 5pm on Tuesday, December 13

You are to work independently on this problem set. You are free to use whatever notes, books, computers, or other reference works you feel are useful. You are also free to consult the course instructor for help, including posting questions to the class email list.

You may *not*, however, consult other students in the class.

Please attach this page to your homework solutions along with your signature, below.

"I have complied with the requirement that I work independently on this problem set. I have not consulted with anyone other than the course instructor in preparing these solutions."

Signature: ______________________________

Print name: ______________________________

1. Imagine a world described by a scalar field ϕ and a fermion field ψ, ruled by the Lagrangian

$$\mathcal{L} = \mathcal{L}^{(\phi)}_{\text{free}} + \mathcal{L}^{(\psi)}_{\text{free}} + \mathcal{L}^{(\phi)}_{\text{self}} + \mathcal{L}_{\text{int}}$$

where the first two terms represent the free fields, the third term is a quartic self energy for the scalar field, and the last term is the minimal interaction between the scalar and the fermion fields.

   a. Write out each of the four terms. Let the scalar and fermion fields have masses μ and m respectively. Use coupling constants λ and f for the scalar self energy and scalar-fermion interaction, respectively.

   b. Draw the lowest order fully connected Feynman diagrams and (schematically) write out the Wick contraction for the following processes:
      - Elastic scattering of two fermions
      - Elastic scattering of a fermion and a scalar
      - Inelastic scattering of two scalars producing a fermion-antifermion pair

   c. Draw the diagram which renormalizes the fermion propagator (to lowest order in f) and determine whether it converges, diverges logarithmically, or diverges faster than logarithmically.

2. (From Zee Exercise I.8.1.) Some authors prefer to write the scalar field in canonical quantization (Eq.I.8.11) without the factor with the square root in the denominator, and to then define the operators $a(\vec{k})$ and $a^\dagger(\vec{k})$ accordingly. Show that for an arbitrary function f

$$\int d^4k\delta(k^2 - m^2)\theta(k^0)f(k^0, \vec{k}) = \int \frac{d^3k}{2\omega_k} f(\omega_k, \vec{k})$$

Argue that $d^3k/2\omega_k$ is therefore a Lorentz invariant measure. [Hint: Lorentz transformations cannot change the sign of k^0.] Thus, the creation and annihilation operators defined by these authors are Lorentz covariant. Work out the commutation relations of these operators.

3. Consider the Dirac Equation for an electron in an electromagnetic field:

$$\left[i\gamma^\mu \left(\partial_\mu - ieA_\mu\right) - m\right]\psi(x) = 0$$

Show that the "charge conjugate" field $\psi_c \equiv \gamma^2\psi^*$ is governed by the same equation, but for an "electron" with the opposite sign charge. Further, show that (See Zee Exercise II.1.8) that the charge conjugate of a left handed field is a right handed field.

4. Use http://www.slac.stanford.edu/library/nobel/ to look up the Physics Nobel Prizes awarded in the following years:

1949, 1957, 1962, 1965, 1972, 1973, 1979, 1982, 1999, 2003, 2004, 2005

In each of these years, the prize was awarded, at least in part, for work in theoretical physics closely connected to quantum field theory.

Choose one of these years and identify the seminal paper(s) for that Prize, or a published version of the laureate's Nobel lecture, and discuss (in one page or less) any ways in which that work is connected to what we covered in class. I encourage you to pick a case which is as close as possible to your own anticipated field of research.