PHYS6530 *Quantum Mechanics III*

Fall 2005 Problem Set #6

Due *in the Physics Department office* on Tuesday, November 22

1. Consider the $N = 2$ scalar field with spontaneous symmetry breaking, written as a complex scalar field as (Zee Eq.IV.1.6):

$$\mathcal{L} = \partial\phi^\dagger\partial\phi + \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2$$

Rewrite $\mathcal{L}$ in terms of two new fields $\chi(x)$ and $\theta(x)$ where $\phi = \rho(x)e^{i\theta(x)}$ and $\rho = v + \chi$ with $v = +\sqrt{\mu^2/2\lambda}$. Thus, derive Zee's Eq.IV.1.7. Identify the mass of the θ and χ fields, their respective potential energy functions, and also their interactions.

2. (See Zee Exercise IV.1.1.) Show explicitly that there are $N - 1$ Nambu-Goldstone bosons in the $G = O(N)$ example given by Eq.(IV.1.2).

3. In Zee's notation, the Goldberger-Treiman relation is written as (Eq.IV.2.7)

$$2m_N F(0) + f g_{\pi NN} = 0$$

Look up values for the four quantities m_N, $F(0)$, f, and $g_{\pi NN}$, and determine the extent to which this relation is satisfied experimentally. Comment on the level of agreement in this case. Include references for the values that you look up.

4. (See Zee Exercise III.1.3.) We would like a little more insight as to how the coupling constant λ depends on the cutoff Λ. Consider the equation for the matrix element $\mathcal{M}$ (Zee Eq.III.1.3), and let the cutoff change to $e^\varepsilon\Lambda$. For $\mathcal{M}$ not to change, show that λ must change by the amount $\delta\lambda = 6\varepsilon C\lambda^2 + \mathcal{O}(\lambda^3)$. Consequently, show that

$$\Lambda\frac{d\lambda}{d\Lambda} = 6C\lambda^2 + \mathcal{O}(\lambda^3)$$

5. (Zee Exercise III.3.1.) Show that in a $(1+1)$ dimensional spacetime the Dirac field ψ has mass dimension $\frac{1}{2}$, and hence the Fermi coupling is dimensionless.