

PHYS6530 *Quantum Mechanics III*

Fall 2005 Problem Set #5

Due at Start of Morning Class on November 9

Introduction

In class, our prototypical “fermions” have been the electron and positron, e^\pm . However, nature appears to have given us two other almost identical kinds of particle/antiparticle pairs, namely the μ^\pm and the τ^\pm . Collectively, these particles are called “leptons”. The only difference seems to be the masses of these particles. As compared to $m_{e^\pm} = 0.511 \text{ MeV}/c^2$, we measure $m_{\mu^\pm} = 105.7 \text{ MeV}/c^2$ and $m_{\tau^\pm} = 1777.0 \text{ MeV}/c^2$.

Reactions with all these particle show excellent agreement with experiment and with quantum electrodynamics calculations. This is an important reason that we believe them all to be the “same” kind of particle. They are distinguished from each other, however, by other properties, and virtual photons cannot couple one particle species to another.

Problem

Calculate the differential cross section $d\sigma/d\Omega$ for the reaction $e^+e^- \rightarrow \ell^+\ell^-$ where ℓ^\pm is a lepton with mass $m \neq m_{e^\pm}$. Do this as we did in class, that is, draw any relevant lowest order Feynman diagrams, and carry out the evaluation of the spin-averaged/summed matrix element-squared. Then, evaluate the necessary phase space integrals and δ -functions to determine $d\sigma/d\Omega$. Do *not* neglect the mass m_{ℓ^\pm} in your calculation.

Also determine the total cross section σ_{tot} by integrating $d\sigma/d\Omega$ over all solid angle.

Work in the e^+e^- center of mass frame when reducing your results to physically measurable quantities. In what follows, $E = E_{e^\pm}$ in this frame, and so $E_{\text{CM}} = 2E$.

For either $\ell = \mu$ or $\ell = \tau$ (or both), compare your calculation with data for

- $d\sigma/d\Omega$ for some energy $E \gg m_\ell$
- σ_{tot} for center of mass energies $E_{\text{CM}} \approx 90 \text{ GeV}$
- σ_{tot} for center of mass energies $E_{\text{CM}} \approx 2m_\ell c^2$

and comment on the level of agreement. If there is significant disagreement, try to identify the reason for it.

One handy place to look up data for such reactions is the Durham reactions data base: <http://durpdg.dur.ac.uk/HEPDATA/HEPDATA.html>