PHYS6530 *Quantum Mechanics III*

Fall 2005 Problem Set #2

Due at Start of Class on September 28

1. (Zee, Ex.I.2.2) Derive the Wick contraction formula for the product of an arbitrary number of operators x_i:

$$\langle x_i x_j \cdots x_k x_\ell \rangle = \sum_{\text{Wick}} \left(A^{-1}\right)_{ab} \cdots \left(A^{-1}\right)_{cd}$$

   As Prof. Zee implores, follow his lead and work out $\langle x_i x_j x_k x_\ell x_m x_n \rangle$ explicitly.

2. (From Zee, Ex.I.3.1) Verify that $D(x)$ decays exponentially for spacelike separation. That is, show that $D(x) \sim e^{-m|\vec{x}|}$ following the arguments in the last paragraph on page 23.

3. (Zee, Ex.I.3.2) Work out the propagator $D(x)$ for a free-field theory in (1+1) dimensional spacetime and study the large x^1 behavior for $x^0 = 0$.

4. (Zee, Ex.I.4.1) Calculate the analog of the inverse square law in a (2+1) dimensional universe, and more generally in a $(D+1)$ dimensional universe.

5. (From Zee, Ex.I.5.1) Write down the most general form for $\sum_a \varepsilon^{(a)}_{\mu\nu}(k)\varepsilon^{(a)}_{\lambda\sigma}(k)$ using symmetry repeatedly. For example, it must be invariant under exchanges such as $\{\mu\nu \leftrightarrow \lambda\sigma\}$. You might end up with something like

$$\begin{aligned} &AG_{\mu\nu}G_{\lambda\sigma} + B(G_{\mu\lambda}G_{\nu\sigma} + G_{\mu\sigma}G_{\nu\lambda}) + C(G_{\mu\nu}k_\lambda k_\sigma + k_\mu k_\nu G_{\lambda\sigma}) \\ &+D(k_\mu k_\lambda G_{\nu\sigma} + k_\mu k_\sigma G_{\nu\lambda} + k_\nu k_\sigma G_{\mu\lambda} + k_\nu k_\lambda G_{\mu\sigma}) + Ek_\mu k_\nu k_\lambda k_\sigma \end{aligned}$$

   with various unknown A, ..., E. Apply $k^\mu \sum_a \varepsilon^{(a)}_{\mu\nu}(k)\varepsilon^{(a)}_{\lambda\sigma}(k) = 0$ and find out what that implies for the constants. Proceeding in this way, derive

$$\sum_a \varepsilon^{(a)}_{\mu\nu}(k)\varepsilon^{(a)}_{\lambda\sigma}(k) = (G_{\mu\lambda}G_{\nu\sigma} + G_{\mu\sigma}G_{\nu\lambda}) - \frac{2}{3}G_{\mu\nu}G_{\lambda\sigma}$$

6. (From Zee, Ex.I.6.1) Putting in the numbers, show that the case $n = 1$ (where n is the number of "extra" dimensions) is already ruled out. For help, see S. Nussinov and R. Schrock, *Phys.Rev.* D59(1999)105002. (You can get this article online using http://prola.aps.org/)

Note: You will not have much difficulty locating outlines for the solutions to problems 2 and 3. You are of course free to use these, but please be explicit of the steps in your calculation. You may need to consult a textbook on complex variables.