PHYS6530 *Quantum Mechanics III*

Fall 2004 Problem Set #7 (Final Problem Set)

Due in the Physics Office (Instructor's Mailbox) by 5pm on Tuesday, December 14

You are to work independently on this problem set. You are free to use whatever notes, books, computers, or other reference works you feel are useful. You are also free to consult the course instructor for help, including posting questions to the class email list.

You may *not*, however, consult other students in the class.

Please attach this page to your homework solutions along with your signature, below.

"I have complied with the requirement that I work independently on this problem set. I have not consulted with anyone other than the course instructor in preparing these solutions."

Signature: _____

Print name: _____

1. *This fixes a mistake that I made during my class presentation of calculating the differential cross section.* Show that

$$\int_{-\infty}^{\infty} f(x)\delta(g(x))dx = \left[\frac{f(x)}{\partial g/\partial x}\right]_{g=0}$$

   Then, use Zee Eq.(C.21) to derive the differential cross section $d\sigma/d\Omega$ for the two-body reaction $p_1 + p_2 \to k_1 + k_2$ (particles labeled by their momenta) in terms of the transition matrix element $|\mathcal{M}|^2$.

2. Show that the Dirac Lagrangian is invariant under the continuous symmetry transformation $\psi \to e^{i\phi\gamma^5}\psi$ if the mass $m = 0$. Derive the corresponding Noether current. How does this current transform under a Lorentz transformation?

3. Imagine a world described by a scalar field ϕ and a fermion field ψ, ruled by the Lagrangian

$$\mathcal{L} = \mathcal{L}^{(\phi)}_{\text{free}} + \mathcal{L}^{(\psi)}_{\text{free}} + \mathcal{L}^{(\phi)}_{\text{self}} + \mathcal{L}_{\text{int}}$$

   where the first two terms represent the free fields, the third term is a quartic self energy for the scalar field, and the last term is the minimal interaction between the scalar and the fermion fields.

   a. Write out each of the four terms. Let the scalar and fermion fields have masses μ and m respectively. Use coupling constants λ and f for the scalar self energy and scalar-fermion interaction, respectively.
   b. Draw the lowest order fully connected Feynman diagrams and (schematically) write out the Wick contraction for the following processes:
      - Elastic scattering of two fermions
      - Elastic scattering of a fermion and a scalar
      - Inelastic scattering of two scalars producing a fermion-antifermion pair
   c. Draw the diagram which renormalizes the fermion propagator (to lowest order in f) and determine whether it converges, diverges logarithmically, or diverges faster than logarithmically.

4. Take our familiar Lagrangian density for a complex relativistic scalar field, namely

$$\mathcal{L} = (\partial_\mu\phi)^\dagger (\partial^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2$$

   and replace ϕ with two real fields $\sigma(x)$ and $\theta(x)$ using $\phi = \sigma e^{i\theta}$. Rewrite the Lagrangian and identify terms, if there exist, which describe free fields $\sigma(x)$ or $\theta(x)$, and the terms which desribe their interactions. Then, make the substitution $m^2 \equiv -\mu^2 > 0$ and rewrite $\mathcal{L}$ using $\sigma(x) = v + \rho(x)$ where $\sigma = v$ represents the mimimum of the "potential energy". Comment once again on the terms governing the behavior of $\rho(x)$ and $\sigma(x)$.

5. Pick one Nobel Prize awarded for work in *theoretical* physics which is closely connected to developments in quantum field theory. Identify the seminal paper(s) for that Prize, or a published version of the laureate's Nobel lecture, and discuss (in one page or less) any ways in which that work is connected to what we covered in class. I encourage you to pick a Prize which is as close as possible to your own anticipated field of research. You might find the web sites http://www.slac.stanford.edu/library/nobel/ or http://nobelprize.org/physics/ particularly useful.