PHYS6530 *Quantum Mechanics III*

Fall 2004 Problem Set #6

Due *in the Physics Department office* on Tuesday, November 23

1. (Zee Exercise III.3.1.) Show that in a $(1+1)$ dimensional spacetime the Dirac field ψ has mass dimension $\frac{1}{2}$, and hence the Fermi coupling is dimensionless.

2. Defy gauge invariance by looking for a function $D_{\mu\nu}(x)$ such that $Q^{\mu\nu}D_{\mu\nu}(x) = \delta^{(4)}(x)$ where $Q^{\mu\nu} = \partial^2\eta^{\mu\nu} - \partial^\mu\partial^\nu$. (See Chapter III.4 of Zee.) Demonstrate that this is not possible.

3. (See Zee Exercise III.5.1.) Obtain the Klein-Gordon equation for a particle in an electrostatic potential (such as that of the nucleus) by the gauge principle of replacing $(\partial/\partial t)$ in Eq.(III.5.2) by $\partial/\partial t - ieA_0$. Show that in the nonrelativistic limit this reduces to the Schrödinger's equation for a particle in an external potential.

4. (See Zee Exercise III.6.1.) Evaluate $\bar{u}(p')(\not{p}'\gamma^\mu + \gamma^\mu\not{p})u(p)$ in two different ways and thus prove Gordon decomposition, i.e. Eq.(III.6.6).

5. (See Zee Exercise IV.1.1.) Show explicitly that there are $N-1$ Nambu-Goldstone bosons in the $G = O(N)$ example given by Eq.(IV.1.2).