PHYS6530 *Quantum Mechanics III*

Fall 2004 Problem Set #4
Due at Start of Morning Class on October 27

You are to work independently on this problem set. You are free to use whatever notes, books, computers, or other reference works you feel are useful. You are also free to consult the course instructor for help, including posting questions to the class email list.

You may *not*, however, consult other students in the class.

Please attach this page to your homework solutions along with your signature, below.

"I have complied with the requirement that I work independently on this problem set. I have not consulted with anyone other than the course instructor in preparing these solutions."

Signature: ______________________

Print name: ______________________

1. Explicitly carry out the integral for the potential energy due to exchange of a field particle:
$$V\left(|\vec{x}|\right) = \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{x}}}{|\vec{k}|^2 + m^2} = \frac{1}{4\pi\,|\vec{x}|} e^{-m|\vec{x}|}$$
For this problem, please be as rigorous with the math as possible.

2. The "propagator" for our scalar field theory is the function $D(x)$ which turns out to satisfy the differential equation $-(\partial^2 + m^2)D(x) = \delta^{(4)}(x)$. Show that
$$iD(x_1 - x_2) = \int D\phi \; e^{i\int d^4x \mathcal{L}_{\rm free}} \phi(x_1)\phi(x_2)$$
where $\mathcal{L}_{\rm free} = \frac{1}{2}\left[(\partial\phi)^2 - m^2\phi^2\right]$. You don't need to be any more rigorous with the math than we are in class, but when you substitute a plausibility argument for rigor, please be as explicit as possible.

3. For our scalar field theory with a $\lambda\phi^4$ "interaction term", consider the $\mathcal{O}(\lambda^2)$ term of the six point Green's function. Draw all Feynman diagrams associated with this term, and write down the Wick Contraction associated with each diagram.

4. In class and in our textbook, we identified 16 linearly independent 4×4 matrices as I, γ^μ, γ^5, $\gamma^\mu\gamma^5$, and $\sigma^{\mu\nu} \equiv \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$. Take the adjoint of these and explicitly show which of the 16 matrices are Hermitan and which are not.

5. For the free Dirac Lagrangian $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$, show that the transformation $\psi \to e^{i\theta}\psi$ leaves the Lagrangian invariant. Use Noether's theorem to derive the corresponding conserved current.

6. (From Zee Exercise II.1.3.) Solve the free Dirac equation in momentum space, namely $(p\!\!\!/ - m)\psi(p) = 0$, explicitly. (You can assume a particle moving with momentum $\vec{p}$ in the z, i.e. "3", direction. Rotational invariance assures you that this is a general solution.) For an electron with $|\vec{p}| \ll m$ show that the "bottom half" of ψ is indeed much smaller than the top. Examine the solution also for $|\vec{p}| \gg m$ and examine what happens to it when multiplied by γ^5.