# PHYS6530 *Quantum Mechanics III*

Fall 2004 Problem Set #3

Due *to the grader* on Monday, October 18

1. For the integral evaluated in Zee's "baby problem", namely

$$\tilde{Z}(J) = \frac{1}{(2\pi/m^2)^{1/2}} \int_{-\infty}^{\infty} dq e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4 + Jq}$$

find the terms of order $\lambda J^0$, $\lambda J^2$, $\lambda J^4$, $\lambda J^6$, $\lambda J^8$, $\lambda J^{10}$, $\lambda^2 J^0$, $\lambda^2 J^2$, $\lambda^2 J^4$, $\lambda^2 J^6$, $\lambda^2 J^8$, and $\lambda^2 J^{10}$. (As soon as you identify the pattern, you can just write the term down without actually having to figure it out.) Draw the diagrams corresponding to at least some of these terms, but including Zee's Figures I.7.2 and I.7.3 with the diagrams he left out of these two figures. (See the Errata, posted on the class web page.)

2. (From Zee Exercise I.7.2.) Derive the expression (I.7.23), the result of applying Feynman rules to the loop diagram shown in Fig.(I.7.10), directly from first principles, namely Eq.(I.7.12). This will lead you to the "pesky" symmetry factor of $\frac{1}{2}$ for this diagram. The calculation is straightforward, but a bit tedious. See the suggestions at the back of the textbook.

3. (From Zee Exercise I.8.3) We have discussed a hermitean (often called real in a minor abuse of terminology) scalar field. Consider instead a complex scalar field (which we actually get to in Chapter I.9) governed by $\mathcal{L} = \partial\phi^\dagger\partial\phi - m^2\phi^\dagger\phi$. Write down the Euler-Lagrange equations of motion treating $\phi$ and $\phi^\dagger$ as independent variables. The nonhermiticity of $\phi$ means that we have to replace Eq.(I.8.11) by

$$\phi(\vec{x}, t) = \int \frac{d^D k}{\sqrt{(2\pi)^D 2\omega_k}} \left[ a(\vec{k}) e^{-i(\omega_k t - \vec{k}\cdot\vec{x})} + b^\dagger(\vec{k}) e^{+i(\omega_k t - \vec{k}\cdot\vec{x})} \right]$$

Show that the canonical commutation relations imply that $(a, a^\dagger)$ and $(b, b^\dagger)$ form two independent sets of creation and annihilation operators. Calculatge $\langle 0|T[\phi(x)\phi^\dagger(0)|0\rangle$. [Hint: $\partial_0\phi$ is conugate to $\phi^\dagger$, not $\phi$,]

4. (Zee Exercise I.8.4) Using the equations of motion, verify that the current $J_\mu = i(\phi^\dagger\partial_\mu\phi - \partial_\mu\phi^\dagger\phi)$ is conserved and that the corresponding charge is given by $Q = \int d^D x J_0(x) = \int d^D k \left[ a^\dagger(\vec{k})a(\vec{k}) - b^\dagger(\vec{k})b(\vec{k}) \right]$. We see that the particle created by $a^\dagger$ (call it the "particle") and the particle created by $b^\dagger$ (call it the "antiparticle") carry opposite charge. We see that $\phi^\dagger$ creates a particle and annihilates an antiparticle, that is, it produces one unit of charge. The field $\phi$ does the opposite. Show that $[Q, \phi(x)] = -\phi(x)$.

5. (From Zee Exercise I.8.6.) Show off your skill in doing integrals by calculating the Casimir force in (3+1)-dimensional spacetime. For help, see M. Kardar and R. Golestanian, *Rev.Mod.Phys.*71(1999)1233, which you can get online through the "Physical Review OnLine Archive" at http://prola.aps.org/, or J. Feinberg, A. Mann, and M. Revzen, *Ann.Phys.*288(2001)103 (note typo in Zee for the page number), which may be found online through Science Direct at http://www.sciencedirect.com/. (Either way, you can find these papers through the RPI library's ejournals web page.) You might also look at Section 5.9 of Huang. See if you can find any experimental papers that perform experimental tests of the Casimir force.