PHYS6520 *Quantum Mechanics II*

Spring 2003 Problem Set #3

Due at Start of Class on February 27

1. (See Merzbacher Exercise 19.7.) Using first order time dependent perturbation theory, and the electric dipole approximation, determine the absorption cross section for the the $1S \rightarrow 2P$ transition in atomic hydrogen. Using the Thomas-Reiche-Kuhn sum rule, determine the fraction of the total cross section represented by this one transition.

2. (Sakurai Problem 5.36.) Derive an expression for the density of free particle states in two dimensions, that is, the two-dimensional analog of

$$\rho(E)dEd\Omega = \left(\frac{L}{2\pi}\right)^3 \left(\frac{mk}{\hbar^2}\right) dEd\Omega$$

with $\vec{k} \equiv \vec{p}/\hbar$ and $E = |\vec{p}|^2/2m$. Your answer should be written as a function of k (or E) times $dEd\phi$, where ϕ is the polar angle that characterizes the momentum direction in two dimensions.

3. (Sakurai Problem 5.38.) Linearly polarized light of angular frequency ω is incident on a one-electron "atom" whose wave function can be approximated by the ground state of a three-dimensional isotropic harmonic oscillator of angular frequency ω_0. Show that the differential cross section for the ejection of a photoelectron is given by

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{4\alpha\hbar^2 k_f^3}{m^2\omega\omega_0}\left[\frac{\pi\hbar}{m\omega_0}\right]^{1/2} \exp\left\{-\frac{\hbar}{m\omega_0}\left[k_f^2 + \left(\frac{\omega}{c}\right)^2\right]\right\} \\ &\times \sin^2\theta\cos^2\phi \exp\left[\left(\frac{2\hbar k_f \omega}{m\omega_0 c}\right)\cos\theta\right]\end{aligned}$$

provided the ejected electron of momentum $\hbar k$ can be regarded as being in a plane-wave state. (The coordinate system used is shown in Sakurai Figure 5.10.)

4. (See Sakurai Problem 7.1.) The scattering Green's function is defined by the relation (Sakurai Eq. 7.1.11)

$$G_+(\vec{x}, \vec{x}') \equiv \frac{\hbar^2}{2m}\left\langle \vec{x} \left| \frac{1}{E - H_0 + i\varepsilon} \right| \vec{x}' \right\rangle$$

Consider the one-dimensional case, with a particle moving from left to right (i.e. towards positive x) with momentum $p = \hbar k$. Determine the form of the scattering Green's function for this case, and compare it to the form in three dimensions, namely Sakurai Eq. 7.1.12. (*Continued on the next page...*)

You need to perform a (simple) contour integration in the complex plane to solve this problem. You may want to consult a textbook on complex variable analysis. In particular, you will need the Cauchy Integral theorem

$$f(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z - z_0} dz$$

for a function $f(z)$ that is analytic within the region bounded by the contour C.

5. Find the differential cross section in Born approximation for the following scattering potential energy functions, and discuss the points mentioned.

   a. A $1/r^2$ potential, i.e. $V = C/r^2$, where C is a constant. Compare the result to that for Coulomb ("Rutherford") scattering, that is a $1/r$ potential, derived in class using the limit of the Yukawa form. (See Sakurai Eq. 7.2.11.) In particular, could this result be obtained classically?

   b. A potential well of radius a, namely

$$V(r) = \begin{cases} -V_0 & r < a \\ 0 & r > a \end{cases}$$

   Explain what one might measure in an experiment to determine the well radius. You would likely find this discussed in a number of textbooks, but a quick search of the literature points (for example) to Dugan, *et al.*, Physical Review C **8**(1973)909.