

1. (Merzbacher Exercise 17.32.) Prove from the Wigner-Eckart theorem and the special case Clebsch-Gordan coefficients given in Merzbacher Eq.(17.62), that the reduced matrix of \vec{J} is

$$\langle \alpha j' || \vec{J} || \alpha j \rangle = \sqrt{j(j+1)} \hbar \delta_{jj'}$$

2. (Merzbacher Problem 17.6.) The magnetic moment operator for a nucleon of mass m_n is $\vec{\mu} = e(g_\ell \vec{L} + g_s \vec{S})/2m_n c$, where $g_\ell = 1$ and $g_s = 5.587$ for a proton, and $g_\ell = 0$ and $g_s = -3.826$ for a neutron. In a central field with an additional spin-orbit interaction, the nucleons move in shells characterized by the quantum numbers ℓ and $j = \ell \pm 1/2$. Calculate the magnetic moment of a single nucleon as a function of j for the two kinds of nucleons, distinguishing the two cases $j = \ell + 1/2$ and $j = \ell - 1/2$. Plot j times the effective gyromagnetic ratio versus j , connecting in each case the points by straight-line segments. (These are called *Schmidt lines*.)
3. Use first order perturbation theory to estimate the effect of the finite size of the proton on the ground state energy level of the hydrogen atom. Assume the proton is a uniformly charged sphere of radius R , and consider the difference between the potential energy in this model, and the idealized potential. Find a value for the proton radius, the size of this energy shift, and compare it to the size of the spin-orbit splitting.
4. (See Sakurai Problem 5.28.) A collection of hydrogen atoms in the ground state are in between the plates of a parallel plate capacitor. The capacitor is turned on suddenly at time $t_0 = 0$, leading to an electric field $\mathcal{E}(t) = \mathcal{E}_0 e^{-t/\tau}$.
- The transition probabilities to which of the various $n = 2$ final states will be nonzero as $t \rightarrow \infty$. Explain your reasoning. Calculate the transition probability for any final states for which the result may be nonzero. (I suggest that you use MAPLE for the radial integrals.)
 - Take the appropriate limits for \mathcal{E}_0 and for τ so that the voltage pulse is a δ -function. Discuss the resulting transition probability(ies).
5. (Merzbacher Exercise 19.10.) Deduce the selection rule for autoionizing transitions from a $2s np$ initial to a $1s kl$ final state. Do the same for transitions from $(2p)^2$ to $1s kl$. Identify the terms in the multipole expansion of the electron-electron Coulomb interaction that are responsible for the transitions.
6. (Merzbacher Problem 19.2.) Calculate the total cross section for photoemission from the K shell as a function of the frequency of the incident light and the frequency of the K -shell absorption edge, assuming that $\hbar\omega$ is much larger than the ionization potential but that nevertheless the photon momentum is much less than the momentum of the ejected electron. Use a hydrogenic wave function for the K shell and plane waves for the continuum states.