

PHYS6520 *Quantum Mechanics II*

Spring 2002 Problem Set #3

Due at Start of Class on February 25

1. A particle of mass m and charge e oscillates in a one-dimensional harmonic oscillator potential with angular frequency ω . An electric field \mathcal{E} is applied.
 - a. Use perturbation theory to calculate the change in the energy eigenvalue for the n th energy level.
 - b. Solve the problem exactly, and compare to the result from part (a).
2. (Sakurai Problem 11, Chapter 5. See also class notes from February 11, 2002.) The Hamiltonian matrix for a two-state system can be written as

$$H = \begin{pmatrix} E_1^0 & \Delta \\ \Delta & E_2^0 \end{pmatrix}$$

Clearly the energy eigenfunctions for the unperturbed states are given by

$$\phi_1^{(0)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \phi_2^{(0)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- a. Solve this problem *exactly* to find the energy eigenfunctions ψ_1 and ψ_2 and the energy eigenvalues E_1 and E_2 .
- b. Assuming that $|\Delta| \ll |E_1^0 - E_2^0|$, solve the same problem using perturbation theory up to first order in the energy eigenfunctions and up to second order in the energy eigenvalues. Compare with the exact results in part (a).
- c. Suppose the two unperturbed energies are “almost degenerate”, that is

$$|E_1^0 - E_2^0| \ll |\Delta|$$

Show that the exact results obtained in part (a) closely resemble what you would expect by applying *degenerate* perturbation theory to this problem with E_1^0 set exactly equal to E_2^0 .

3. (See Merzbacher Exercise 18.1) The Hamiltonian of a rigid rotor in a magnetic field perpendicular to the x axis is of the form $A\vec{L}^2 + BL_z + CL_y$, if the term quadratic in the field is neglected. Assuming $B \gg C$, use perturbation theory to get approximate energy eigenvalues.

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4. (See Merzbacher Exercise 18.9.) Relativistically, the kinetic energy of an electron is given by

$$T_p = \left(\vec{p}^2 c^2 + m^2 c^4\right)^{1/2} - mc^2 = \frac{\vec{p}^2}{2m} - \frac{(\vec{p}^2)^2}{8m^3 c^2} + \dots$$

Treating the second term as a perturbation, calculate the first order shift in the ground state of a one-electron “hydrogen” atom with atomic number Z . Briefly discuss the effect for large Z .

5. This problem discusses the Zeeman Effect, namely the splitting of atomic energy levels in the presence of a magnetic field $\vec{B} = B\hat{z}$. We will consider an atom with a single valence electron in an $\ell = 1$ state of orbital angular momentum, for a specific value of the radial quantum number n . The perturbation Hamiltonian is

$$H_B = \frac{\mu_B B}{\hbar} (L_z + 2S_z)$$

where $\mu_B = 5.788 \times 10^{-5}$ eV/T is the Bohr magneton.

- Use Clebsch-Gordan coefficients to write the six states $|jm\rangle$ in terms of the six possible combinations of the quantum numbers of orbital and spin angular momentum.
- The “spin-orbit” interaction H_{SO} is proportional to the operator $\vec{L} \cdot \vec{S}$. Explain how this interaction *only partially* lifts the degeneracy between the six states $|jm\rangle$. From class notes or the textbook, or some other source, estimate the size of the spin-orbit splitting (in eV) for the $n = 2$ states of a hydrogen atom.
- Explain why degenerate perturbation theory is needed to calculate the effect of H_B using the $|jm\rangle$ basis. Determine the perturbed energies of all six states as a function of magnetic field B , so long as $\mu_B B$ is much less than the spin-orbit splitting.
- Sketch the energies of all six states as a function of B for a hydrogen atom. (See Figure 18.1 in Merzbacher.) Include numerical values for the vertical and horizontal axes. Give a value of B where you are certain that the this perturbation approximation breaks down.