

1. The following parts are unrelated to each other, but they count together as one problem.
 - (a) Show that under a unitary transformation U , a state vector $|\alpha\rangle$ preserves its normalization.
 - (b) (Merzbacher Exercise 10.6.) Show that if A and B are commuting operators, then B has no nonvanishing matrix element between eigenstates corresponding to different eigenvalues of A .
 - (c) (See Merzbacher Exercise 14.9.) Suppose A and B are operators that correspond to observables which are constants of the motion. Show that the operator $i[A, B]$ also corresponds to an observable which is a constant of the motion, regardless of whether or not A and B commute. Illustrate this with the orbital angular momentum operators L_x and L_y .
2. The spherical Bessel functions $j_\ell(z)$ can be defined as (Merzbacher Eq.12.19)

$$j_\ell(z) = \frac{1}{2i^\ell} \int_{-1}^1 e^{izs} P_\ell(s) ds$$

Use this definition to prove that, for a real variable x , $j_{\ell'}(x)$ and $j_\ell(x)$ are orthogonal to each other unless $\ell' = \ell$. In the case where $\ell' = \ell$, derive the normalization factor. You may use whatever relations you find in Merzbacher, but cite the appropriate equation number. (*Hint:* As shown in the appendix, Eq.A.8, $\delta(x - u) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik(x-u)} dk$.)

3. For the one dimensional simple harmonic oscillator, write down the Schrödinger equation in the *momentum* representation. Give the momentum space wave function $\phi(p) = \langle p|n\rangle$ for the energy eigenstate $|n\rangle$. Justify your answers in few words.
4. (Merzbacher Problem 12.5.) Assuming the eigenfunctions for the hydrogen atom to be of the form $r^\beta e^{-\alpha r} Y_\ell^m$, with undetermined parameters α and β , solve the Schrödinger equation. Are all eigenfunctions and eigenvalues obtained in this way?
5. (Merzbacher Problem 14.6.) A linear harmonic oscillator, with energy eigenstate $|n\rangle$, is subjected to a time-dependent interaction between the ground state $|0\rangle$ and the first excited state:

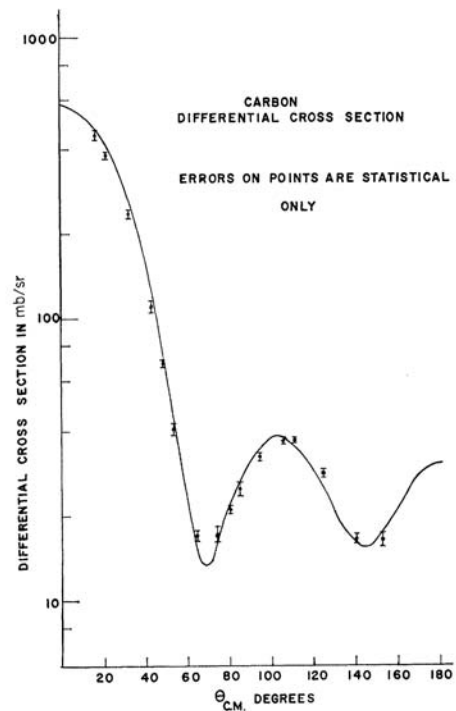
$$V(t) = F(t)|1\rangle\langle 0| + F^*(t)|0\rangle\langle 1|$$

- (a) Derive the coupled equations of motion for the probability amplitudes $\langle n|\Psi(t)\rangle$.
- (b) If $F(t) = \sqrt{2\hbar\omega}\eta(t)$, obtain the energy eigenvalues and the stationary states for $t > 0$. (Recall the definition of the Heaviside step function $\eta(t)$ in Merzbacher Chapter 14, Section 6.)
- (c) If the system is in the ground state of the oscillator before $t = 0$, calculate $\langle n|\Psi(t)\rangle$ for $t > 0$.

6. (See Merzbacher Exercise 13.9.)

- (a) Use the Born Approximation to calculate the differential cross section for scattering of a particle of mass m and momentum $\hbar k$ from a three dimensional square well of depth V_0 and radius a .
- (b) Make a plot of the the differential cross section as a function of qa , where $\hbar q$ is the magnitude of the momentum transfer to the target from the incident particle. The plot should cover at least the values $0 \leq qa \leq 10$.

- (c) The figure on the right shows experimental data on the scattering of 14 MeV neutrons from carbon nuclei. The data is taken from A.J. Frasca, et al., *Physical Review*, v.144 (1966) pg.854. Compare your plot in (b) to this data and use it to estimate the radius and effective well depth for a carbon nucleus. Give reasons why or why not this is a good estimate of these nuclear paramters.



7. (Merzbacher Exercise 15.24.) If the state of a quantum system is given by a density operator

$$\rho = p_1 |\Psi_1\rangle \langle \Psi_1| + p_2 |\Psi_2\rangle \langle \Psi_2|$$

where $|\Psi_{1,2}\rangle$ are two non-orthogonal normalized state vectors, show that the eigenvalues of the density operator are

$$\bar{p}_{1,2} = \frac{1 \pm \sqrt{1 - 4p_1 p_2 (1 - |\langle \Psi_1 | \Psi_2 \rangle|^2)}}{2}$$