PHYS6510 *Quantum Mechanics I*

Fall 2001 Problem Set #6

Due on *Tuesday* November 20, *at 5pm to the grader*

1. (Merzbacher Exercise 13.2.) Consider a 10 MeV α particle passing through a 1 μm thick alumninum foil. Calculate the probability of Rutherford scattering for the α particle through an angle greater than 10^{-2} radians in the laboratory frame. Use the result to estimate the importance of multiple scattering when calculating the total scattering probability in this example.

2. (Merzbacher Exercise 13.4.) Verify that the Green's functions

$$G_{\pm}(\vec{r},\vec{r}') = \frac{\exp\left(\pm ik|\vec{r}-\vec{r}'|\right)}{|\vec{r}-\vec{r}'|}$$

   are solutions to the inhomogeneous Green's function equation

$$(\nabla^2 + k^2)G(\vec{r},\vec{r}') = -4\pi\delta(\vec{r}-\vec{r}')$$

3. (Merzbacher Exercise 13.10.) Obtain the differential scattering cross section in the Born approximation for the potential

$$V(r) = -V_0 e^{-r/a} \qquad\qquad (V_0 > 0)$$

   What is the criterion in this case for the approximation to be valid?

4. This problem concerns the *optical theorem* which relates the forward scattering amplitude, i.e. $f(\theta,\phi)$ for $\theta = 0$, to the total scattering cross section:

$$\sigma = \frac{4\pi}{k}\mathrm{Im} f_k(\theta = 0)$$

   (a) (Merzbacher Exercise 13.14.) For the case of a spherically symmetric potential, show that the partial wave expansion for the scattering amplitude, Merzbacher, Eq. (13.70),

$$f_k(\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell(k)}\sin\delta_\ell(k)P_\ell(\cos\theta)$$

   is related to the partial wave expansion for the total cross section, Merzbacher, Eq. (13.72),

$$\sigma = \frac{4\pi}{k}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell$$

   by the optical theorem.

(b) Given a real, central potential $V(r)$, and the asymptotic form of the wave function

$$\psi_k(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + f_k(\theta, \phi)\frac{\exp(ikr)}{r}$$

use the continuity equation for the conservation of probability, and Gauss' theorem, to derive the optical theorem.

5. (See Merzbacher Exercise 13.15.) Calculate and make a polar plot of the differential scattering cross section from a hard sphere for $ka = 1/10$, $1/2$, and 10, using the first three partial waves, i.e. $\ell = 0$, 1, and 2. What total cross section do you expect? Compare to the differential cross section and use the result to estimate the approximate accuracy of your calculation.

6. Reproduce Merzbacher Figures 13.7 and 13.8. That is, first calculate and plot versus ka, the S, P, and D phase shifts for scattering from a square well of radius a and depth V_0 with $(2mV_0a^2/\hbar^2)^{1/2} = 6.2$. Normalize the phase shifts to approach zero as the energy $E \to \infty$. Secondly, plot the cross section for S-wave and P-wave scattering, also as a function of ka. Compare the cross section plots to the corresponding phase shift plots.