PHYS6510 *Quantum Mechanics I*

Fall 2001 Problem Set #1

Due at Start of Class on September 10

1. (Merzbacher 1.1) To what velocity would an electron have to be slowed down, if its wavelength is to be 1 meter? Repeat the calculation for a neutron. Are matter waves of macroscopic dimensions a real possibility?

2. Unpolarized light of intensity I is incident on a series of three polarizing filters. The first and the third filters are oriented at 90° with respect to each other. The orientation of the middle filter rotates at (angular) frequency ω, and the intensity of the light emerging from the trio of filters varies in time accordingly.

   a. With what frequency does the emerging light intensity vary?

   b. What is the maximum intensity, in terms of I, of the emerging light?

3. (Sakurai 1.1) Prove the following, for operators A, B, C, and D:

$$[AB, CD] = -AC\{D, B\} + A\{C, B\}D - C\{D, A\}B + \{C, A\}DB$$

4. (Sakurai 1.7) Consider a ket space spanned by the eigenkets $\{|a'\rangle\}$ of a Hermitian operator A. There is no degeneracy.

   a. Prove that

$$\prod_{a'} (A - a')$$

   is the null operator.

   b. What is the significance of

$$\prod_{a'' \neq a'} \frac{(A - a'')}{(a' - a'')}$$

   c Illustrate (a) and (b) using A set equal to S_z of a spin $\frac{1}{2}$ system.

5. (Sakurai 1.8) Using the orthonormality of $|+\rangle$ and $|-\rangle$, prove that

$$[S_i, S_j] = i\varepsilon_{ijk}\hbar S_k, \qquad \{S_i, S_j\} = \left(\frac{\hbar^2}{2}\right)\delta_{ij},$$

   where

$$\begin{aligned} S_x &= \frac{\hbar}{2}(|+\rangle\langle-| + |-\rangle\langle+|) \\ S_y &= \frac{i\hbar}{2}(-|+\rangle\langle-| + |-\rangle\langle+|) \\ S_z &= \frac{\hbar}{2}(|+\rangle\langle+| - |-\rangle\langle-|) \end{aligned}$$

6. (Sakurai 1.18a,b)

   a. The simplest way to derive the Schwarz inequality goes as follows. First, observe

   $$(\langle\alpha| + \lambda^\star \langle\beta|) \cdot (|\alpha\rangle + \lambda|\beta\rangle) \geq 0$$

   for any complex number λ; then choose λ in such a way that the preceding inequality reduces to the Schwarz inequality. Do this.

   b. Show that the equality sign in the generalized uncertainty relation holds if the state in question satisfies

   $$\Delta A|\alpha\rangle = \lambda \Delta B|\alpha\rangle$$

   with λ purely *imaginary*.

7. (Sakurai 1.19)

   a. Compute

   $$\left\langle (\Delta S_x)^2 \right\rangle \equiv \langle S_x^2 \rangle - \langle S_x \rangle^2$$

   where the expectation value is taken for the S_z+ state. Using your result, check the generalized uncertainty relation

   $$\left\langle (\Delta A)^2 \right\rangle \left\langle (\Delta B)^2 \right\rangle \geq \frac{1}{4} \left| \langle [A, B] \rangle \right|^2$$

   with $A \to S_x$ and $B \to S_y$.

   b. Check the uncertainty relation with $A \to S_x$ and $B \to S_y$ for the S_x+ state.