1. Section 4.2, Page 289, question 24.

(a)Given A or B is singular :

By theorem 2, we have det(AB) = det(A)det(B) . By theorem 3 and the given condition we have either det(A) = 0 or det(B) = 0. Hence, det(AB) = 0 and by theorem 3, that makes AB singular.

(b)

Given AB is singular, by theorem 3, we know that det(AB) = 0. Furthermore, by theorem 2 we have det(AB) = det(A)det(B). To get det(AB) = 0, det(A)det(B) must be zero, which implies that at least one of the matrices (A or B) has a determinant of zero, implying that it is singular by theorem 2.

2. Section 4.3, Page 297, question 18.

$$\det(B) = \begin{vmatrix} d & f & e \\ a & c & b \\ g & i & h \end{vmatrix} \underset{=}{\overset{R_1 \leftrightarrow R_2}{\rightarrow}} -\begin{vmatrix} a & c & b \\ d & f & e \\ g & i & h \end{vmatrix} \underset{=}{\overset{C_2 \leftrightarrow C_3}{\rightarrow}} \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -(-2) = \boxed{2}$$

3. Section 4.3, Page 297, question 22.

$$\begin{vmatrix} 2 & 2 & 4 & 4 \\ 1 & 1 & 3 & 3 \\ 1 & 0 & 2 & 1 \\ 4 & 1 & 3 & 2 \end{vmatrix} \underset{=}{\overset{C_3 - C_4}{\rightarrow}} \begin{vmatrix} 2 & 2 & 0 & 4 \\ 1 & 1 & 0 & 3 \\ 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 2 \end{vmatrix} \underset{=}{\overset{R_4 - R_3}{\rightarrow}} \begin{vmatrix} 2 & 2 & 0 & 4 \\ 1 & 1 & 0 & 3 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 \end{vmatrix} = 1(-1)^{3+3}\begin{vmatrix} 2 & 2 & 4 \\ 1 & 1 & 3 \\ 3 & 1 & 1 \end{vmatrix} \underset{=}{\overset{R_1 - 2R_2}{\rightarrow}} \begin{vmatrix} 0 & 0 & -2 \\ 1 & 1 & 3 \\ 3 & 1 & 1 \end{vmatrix} =$$

$$-2(-1)^{1+3}\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -2(1-3) = \boxed{4}$$

4. Section 4.4, Page 305, question 8.

$$p(\lambda) = \begin{vmatrix} -2-\lambda & -1 & 0 \\ 0 & 1-\lambda & 1 \\ -2 & -2 & -1-\lambda \end{vmatrix} = (-1)^{2+2}(1-\lambda)\begin{vmatrix} -2-\lambda & 0 \\ -2 & -1-\lambda \end{vmatrix} + (-1)^{2+3}(1)\begin{vmatrix} -2-\lambda & -1 \\ -2 & -2 \end{vmatrix} =$$

$$-(1-\lambda)\begin{vmatrix} \lambda+2 & 0 \\ 2 & \lambda+1 \end{vmatrix} + \begin{vmatrix} \lambda+2 & 1 \\ 2 & 2 \end{vmatrix} = (\lambda-1)((\lambda+2)(\lambda+1)-0) + (2(\lambda+2)-2) = (\lambda-1)(\lambda+2)(\lambda+1) + (2(\lambda+1)) =$$

$$(\lambda+1)((\lambda-1)(\lambda+2)+2) = (\lambda+1)(\lambda^2+\lambda-2+2) = (\lambda+1)(\lambda^2+\lambda) = \lambda(\lambda+1)^2$$

Eigenvalues when $p(\lambda) = 0 \therefore$ *Eigenvalues are* : $\boxed{\begin{array}{l} \lambda = 0 \text{ with algebraic multiplicity of 1.} \\ \lambda = -1 \text{ with algebraic multiplicity of 2.} \end{array}}$

5. Section 4.4, Page 305, question 18, part (a) only.

Given : $H\mathbf{x} = \lambda\mathbf{x}, \mathbf{x} \neq 0$

$q(H)\mathbf{x} = (H^3 - 2H^2 - H + 2I)\mathbf{x} = H^3\mathbf{x} - 2H^2\mathbf{x} - H\mathbf{x} + 2I\mathbf{x} \Rightarrow \lambda^3\mathbf{x} - 2\lambda^2\mathbf{x} - \lambda\mathbf{x} + 2\mathbf{x}$ *by part* 1 *of theorem* 11.

$\boxed{\textit{Hence an eigenvalue for } q(H) \textit{ is } \lambda^3 - 2\lambda^2 - \lambda + 2 = q(\lambda) \text{ which is what we set out to prove.}}$

6. Section 4.4, Page 306, question 28.

$$\mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \mathbf{x}_1 = A\mathbf{x}_0 = \begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ 1 \end{bmatrix} \qquad \mathbf{x}_2 = A\mathbf{x}_1 = \begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -7 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ -7 \\ 17 \end{bmatrix}$$

$$\mathbf{x}_3 = A\mathbf{x}_2 = \begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ -7 \\ 17 \end{bmatrix} = \begin{bmatrix} 17 \\ -23 \\ 33 \end{bmatrix} \qquad \mathbf{x}_4 = A\mathbf{x}_3 = \begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix} \begin{bmatrix} 17 \\ -23 \\ 33 \end{bmatrix} = \begin{bmatrix} 41 \\ -39 \\ 81 \end{bmatrix}$$

$$\mathbf{x}_5 = A\mathbf{x}_4 = \begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix} \begin{bmatrix} 41 \\ -39 \\ 81 \end{bmatrix} = \begin{bmatrix} 81 \\ -87 \\ 161 \end{bmatrix}$$

$$\beta_0 = \frac{\mathbf{x}_0^T \mathbf{x}_1}{\mathbf{x}_0^T \mathbf{x}_0} = \frac{(1)(1) + (1)(-7) + 1(1)}{(1)(1) + (1)(1) + (1)(1)} = \boxed{-\frac{5}{3}} \qquad \beta_1 = \frac{\mathbf{x}_1^T \mathbf{x}_2}{\mathbf{x}_1^T \mathbf{x}_1} = \frac{(1)(9) + (-7)(-7) + 1(17)}{(1)(1) + (-7)(-7) + (1)(1)} = \boxed{\frac{75}{51}}$$

$$\beta_2 = \frac{\mathbf{x}_2^T \mathbf{x}_3}{\mathbf{x}_2^T \mathbf{x}_2} = \frac{(9)(17) + (-7)(-23) + (17)(33)}{(9)(9) + (-7)(-7) + (17)(17)} = \boxed{\frac{875}{419}} \qquad \beta_3 = \frac{\mathbf{x}_3^T \mathbf{x}_4}{\mathbf{x}_3^T \mathbf{x}_3} = \frac{(17)(41) + (-23)(-39) + (33)(81)}{(17)(17) + (-23)(-23) + (33)(33)} = \boxed{\frac{4267}{1907}}$$

$$\beta_4 = \frac{\mathbf{x}_4^T \mathbf{x}_5}{\mathbf{x}_4^T \mathbf{x}_4} = \frac{(41)(81) + (-39)(-87) + (81)(161)}{(41)(41) + (-39)(-39) + (81)(81)} = \boxed{\frac{19755}{9763}}$$