1. Section 1.5, Page 59, question 48.

$x_1 = x_3 + x_5 + 2x_6,\quad x_2 = -2x_3 - x_5 - 2x_6,\quad x_4 = -x_5 - x_6$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} x_3 + x_5 + 2x_6 \\ -2x_3 - x_5 - 2x_6 \\ x_3 \\ -x_5 - x_6 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} x_3 \\ -2x_3 \\ x_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} x_5 \\ -x_5 \\ 0 \\ -x_5 \\ x_5 \\ 0 \end{bmatrix} + \begin{bmatrix} 2x_6 \\ -2x_6 \\ 0 \\ -x_6 \\ 0 \\ x_6 \end{bmatrix} = \boxed{x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 2 \\ -2 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}}$$

2. Section 1.6, Page 69, question 30.

Let:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},\; B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \text{ Because these are symmetrical}: A = A^T = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix},\; B = B^T = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}$$

Given $(AB)^T = B^T A^T = BA$, *to make AB not symmetric*, (*thus* $AB \neq (AB)^T$), *we must find matrices A and B such that matrix multiplication is not commutative*($AB \neq BA$).

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{12} + a_{12}b_{22} \\ a_{12}b_{11} + a_{22}b_{12} & a_{12}b_{12} + a_{22}b_{22} \end{bmatrix} \quad BA = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{12} & b_{11}a_{12} + b_{12}a_{22} \\ b_{12}a_{11} + b_{22}a_{12} & a_{12}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Examining the symmetry of each matrix or showing the inequality of the above matrices both lead to:

$$a_{11}b_{12} + a_{12}b_{22} \neq a_{12}b_{11} + a_{22}b_{12} \Rightarrow a_{11}b_{12} - a_{22}b_{12} \neq a_{12}b_{11} - a_{12}b_{22} \Rightarrow b_{12}(a_{11} - a_{22}) \neq a_{12}(b_{11} - b_{22})$$

Substituting one simple set of numbers into the above equations, can yield:

$1(2-1) \neq 2(2-1)$ *which implies that*

$\boxed{A = \begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix},\; B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}}$ *is one set of matrices that satisfies the conditions we are looking for.*

3. Section 1.6, Page 69, question 42.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},\; B = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix},\; C = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$(a) A^T + B = C \Rightarrow A^T = C - B = C + -B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \left(-\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} \right) = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} -1 & -3 \\ -1 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = A^T \Rightarrow \boxed{A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}}$$

$$(b) A^T B = C \Rightarrow \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \Rightarrow \begin{matrix} a_{11} & +a_{21} & & & = 2 \\ 3a_{11} & +4a_{21} & & & = 3 \\ & & a_{12} & +a_{22} & = 4 \\ & & 3a_{12} & +4a_{22} & = 5 \end{matrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 3 & 4 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 3 & 4 & 5 \end{bmatrix} \overset{R_2 - 3R_1}{\Rightarrow} \begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 3 & 4 & 5 \end{bmatrix} \overset{R_1 - R_2}{\Rightarrow} \begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 3 & 4 & 5 \end{bmatrix} \underset{R_4 - 3R_3}{\Rightarrow} \begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 1 & -7 \end{bmatrix} \underset{R_3 - R_4}{\Rightarrow}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 11 \\ 0 & 0 & 0 & 1 & -7 \end{bmatrix} \Rightarrow \begin{matrix} a_{11} & & & & 5 \\ & a_{21} & & & -3 \\ & & a_{12} & & = 11 \\ & & & a_{22} & = -7 \end{matrix} \Rightarrow \boxed{A = \begin{bmatrix} 5 & 11 \\ -3 & -7 \end{bmatrix}}$$

$$(c) B\mathbf{C}_1 = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \boxed{\begin{bmatrix} 14 \\ 18 \end{bmatrix}} \qquad \mathbf{B}_1^T C = [1 \quad 1] \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \boxed{[6 \quad 8]}$$

$$(B\mathbf{C}_1)^T \mathbf{C}_2 = \left(\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right)^T \begin{bmatrix} 3 \\ 5 \end{bmatrix} = [14 \quad 18] \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \boxed{[132]}$$

$$\| C\mathbf{B}_2 \| = \left\| \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 18 \\ 32 \end{bmatrix} \right\| = \sqrt{[18 \quad 32] \begin{bmatrix} 18 \\ 32 \end{bmatrix}} = \sqrt{[18 \quad 32] \begin{bmatrix} 18 \\ 32 \end{bmatrix}} = \sqrt{1348} = \boxed{2\sqrt{337}}$$

4. Section 1.7, Page 79, question 30.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ a \end{bmatrix}$$

To make the system linearly dependent, $V\mathbf{x} = \mathbf{0}$ *must not be the only solution to the system:*

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 2 & a & 0 \end{bmatrix} \begin{matrix} \Rightarrow \\ R_2 - 2R_1 \\ R_3 - R_1 \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & a & 0 \end{bmatrix} \begin{matrix} R_1 - R_2 \\ \Rightarrow \\ R_3 - R_2 \end{matrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & a-1 & 0 \end{bmatrix}$$

For this system to be consistent, $a-1$ *must equal* 0. *Thus, when* $\boxed{a = 1}$, *the vectors are linearly dependent.*

5. Section 1.7, Page 79, question 50.

Since we are given that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent in R^m, we know that $k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k_3\mathbf{v}_3 = \theta$ where k_1, k_2, k_3 are not simultaneously zero. Looking at the linear dependence of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ leads to investigating $k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k_3\mathbf{v}_3 + k_4\mathbf{v}_4 = \theta$. If we set $k_4 = 0$, we have the original relationship and since k_1, k_2, k_3 are not simultaneously zero, we can conclude that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is also linear dependent.