1. Section 1.1, Page 13, question 18.

Since $2[x_1 + 3x_2 - 2x_3 = -1] = [2x_1 + 6x_2 - 4x_3 = -2]$,

it is true that $2E_1 = E_2$ and thus $\boxed{\text{the given system yields two coincident planes (ie. the same plane.)}}$

2. Section 1.1, Page 14, question 42.

METHOD 1:

$$x_1^2 - 2x_1 + x_2^2 = 3$$

$$x_1^2 \quad - x_2^2 = 1 \qquad \textit{Add Equations}:$$

$$2x_1^2 - 2x_1 = 4 \Rightarrow x_1^2 - x_1 - 2 = 0 \Rightarrow (x_1 - 2)(x_1 + 1) = 0 \Rightarrow x_1 = -1,\ 2$$

Since $x_2^2 = x_1^2 - 1$, *when* $x_1 = -1$, $x_2^2 = (-1)^2 - 1 = 0$ *and when* $x_1 = 2$, $x_2^2 = (2)^2 - 1 \Rightarrow x_2 = \pm\sqrt{3}$

$$\boxed{(x_1, x_2) = \{(-1,0), (2,\sqrt{3}), (2,-\sqrt{3})\}}$$

METHOD 2:

$$x_1^2 - 2x_1 + x_2^2 = 3$$

$$x_1^2 \quad - x_2^2 = 1$$

$\mathbf{E}_2 - \mathbf{E}_1$:

$$x_1^2 - 2x_1 + x_2^2 = 3$$

$$2x_1 - 2x_2^2 = -2$$

$\mathbf{E}_1 + \mathbf{E}_2$:

$$x_1^2 \quad - x_2^2 = 1$$

$$2x_1 - 2x_2^2 = -2$$

$\frac{1}{2}\mathbf{E}_2$:

$$x_1^2 \quad - x_2^2 = 1$$

$$x_1 \quad - x_2^2 = -1$$

$$x_1^2 = 1 + x_2^2$$

$$x_1 = x_2^2 - 1 \therefore$$

$$(x_2^2 - 1)^2 = 1 + x_2^2 \Rightarrow x_2^4 - 2x_2^2 + 1 = 1 + x_2^2 \Rightarrow x_2^4 = 3x_2^2 \Rightarrow x_2 = 0, \pm\sqrt{3}$$

Since $x_1 = x_2^2 - 1$, *when* $x_2 = \pm\sqrt{3}$, $x_1 = 3 - 1 = 2$ *and when* $x_2 = 0$, $x_1 = 0 - 1 = -1$

$$\boxed{(x_1, x_2) = \{(-1,0), (2,\sqrt{3}), (2,-\sqrt{3})\}}$$

3. Section 1.2, Page 27, question 30.

$$\begin{array}{l} x_1 + x_2 - x_5 = 1 \\ x_2 + 2x_3 + x_4 + 3x_5 = 1 \\ x_1 - x_3 + x_4 + x_5 = 0 \end{array} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 1 & 0 & -1 & 1 & 1 & 0 \end{bmatrix} \underset{R_3 - R_1}{\Rightarrow} \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 0 & -1 & -1 & 1 & 2 & -1 \end{bmatrix} \underset{\substack{R_1 - R_2 \\ R_3 + R_2}}{\Rightarrow}$$

$$\begin{bmatrix} 1 & 0 & -2 & -1 & -4 & 0 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 0 & 0 & 1 & 2 & 5 & 0 \end{bmatrix} \underset{\substack{R_1 + 2R_3 \\ R_2 - 2R_3}}{\Rightarrow} \begin{bmatrix} 1 & 0 & 0 & 3 & 6 & 0 \\ 0 & 1 & 0 & -3 & -7 & 1 \\ 0 & 0 & 1 & 2 & 5 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_1 + 3x_4 + 6x_5 = 0 \\ x_2 - 3x_4 - 7x_5 = 1 \\ x_3 + 2x_4 + 5x_5 = 0 \end{array}$$

$$\boxed{x_1 = -3x_4 - 6x_5, \quad x_2 = 1 + 3x_4 + 7x_5, \quad x_3 = -2x_4 - 5x_5}$$

4. Section 1.2, Page 27, question 36.

$$\begin{aligned} x_1 \ + 2x_2 &= -3 \\ ax_1 \ - 2x_2 &= 5 \end{aligned} \Rightarrow \begin{bmatrix} 1 & 2 & -3 \\ a & -2 & 5 \end{bmatrix} \underset{R_2 - aR_1}{\Rightarrow} \begin{bmatrix} 1 & 2 & -3 \\ 0 & -2-2a & 5+3a \end{bmatrix} \therefore Let \ \ -2-2a=0 \ \ for \ \ no \ \ solution$$

$$-2-2a=0 \Rightarrow -2(1+a)=0 \Rightarrow 1+a=0 \Rightarrow a=-1 \quad Since \ \ 5+3a \neq 0 \quad when \ \ a=-1 \therefore$$

$$\boxed{When \ \ a=-1, \ \ the \ \ system \ \ has \ \ no \ \ solution.}$$

5. Section 1.2, Page 28, question 52.

$$\begin{aligned} x_1 \ + x_2 \ + x_3 &= 34 \\ x_1 \ + x_2 \qquad &= 7 \\ x_2 \ + x_3 &= 22 \end{aligned} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 34 \\ 1 & 1 & 0 & 7 \\ 0 & 1 & 1 & 22 \end{bmatrix} \underset{R_2 - R_1}{\Rightarrow} \begin{bmatrix} 1 & 1 & 1 & 34 \\ 0 & 0 & -1 & -27 \\ 0 & 1 & 1 & 22 \end{bmatrix} \underset{R_3 \leftrightarrow R_2}{\Rightarrow} \begin{bmatrix} 1 & 1 & 1 & 34 \\ 0 & 1 & 1 & 22 \\ 0 & 0 & -1 & -27 \end{bmatrix} \underset{R_1 - R_2}{\Rightarrow}$$

$$\begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 1 & 22 \\ 0 & 0 & -1 & -27 \end{bmatrix} \underset{-R_3}{\Rightarrow} \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 1 & 22 \\ 0 & 0 & 1 & 27 \end{bmatrix} \underset{R_2 - R_3}{\Rightarrow} \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 27 \end{bmatrix} \Rightarrow \begin{aligned} x_1 \qquad\qquad &= 12 \\ + x_2 \qquad &= -5 \\ + x_3 &= 27 \end{aligned}$$

$$\boxed{x_1 = 12, \quad x_2 = -5, \quad x_3 = 27}$$

6. Section 1.3, Page 38, question 24.

(a)

$$\begin{aligned} x_1 \ + 3x_2 \ - x_3 &= b_1 \\ x_1 \ + 2x_2 \qquad &= b_2 \\ 3x_1 \ + 7x_2 \ - x_3 &= b_3 \end{aligned} \Rightarrow \begin{bmatrix} 1 & 3 & -1 & b_1 \\ 1 & 2 & 0 & b_2 \\ 3 & 7 & -1 & b_3 \end{bmatrix} \underset{\substack{R_2 - R_1 \\ R_3 - 3R_1}}{\Rightarrow} \begin{bmatrix} 1 & 3 & -1 & b_1 \\ 0 & -1 & 1 & b_2 - b_1 \\ 0 & -2 & 2 & b_3 - 3b_1 \end{bmatrix} \underset{-R_2}{\Rightarrow} \begin{bmatrix} 1 & 3 & -1 & b_1 \\ 0 & 1 & -1 & b_1 - b_2 \\ 0 & -2 & 2 & b_3 - 3b_1 \end{bmatrix} \underset{\substack{R_1 - 3R_2 \\ R_3 + 2R_2}}{\Rightarrow}$$

$$\begin{bmatrix} 1 & 0 & 2 & 3b_2 - 2b_1 \\ 0 & 1 & -1 & b_1 - b_2 \\ 0 & 0 & 0 & b_3 - b_1 - 2b_2 \end{bmatrix}$$

$$\boxed{To \ \ be \ \ consistent, \ b_3 - b_1 - 2b_2 \ \ must \ \ equal \ \ zero.}$$

(b)

$(i) 3-1-2(1)=0 \therefore System \ \ is \ \ consistent$

$$\begin{bmatrix} 1 & 0 & 2 & 3(1)-2(1) \\ 0 & 1 & -1 & 1-1 \\ 0 & 0 & 0 & 3-1-2(1) \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \boxed{x_1 = 1 - 2x_3, \quad x_2 = x_3}$$

$(ii) -1-1-2(0) = -2 \neq 0 \therefore System \ \ is \ \ inconsistent$

$(iii) 2-0-2(1)=0 \therefore System \ \ is \ \ consistent$

$$\begin{bmatrix} 1 & 0 & 2 & 3(1)-2(0) \\ 0 & 1 & -1 & 0-1 \\ 0 & 0 & 0 & 2-0-2(1) \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \boxed{x_1 = 3 - 2x_3, \quad x_2 = x_3 - 1}$$