1. Section 14.3, Page 1039, question 12.

$\mathbf{F}(x,y) = ye^{xy}\mathbf{i} + xe^{xy}\mathbf{j}$

$\frac{\partial}{\partial y}(ye^{xy}) = xye^{xy} + e^{xy}$

$\frac{\partial}{\partial x}(xe^{xy}) = xye^{xy} + e^{xy}$

$\therefore \mathbf{F}$ is conservative.

$\int ye^{xy}dx = e^{xy} + g(y)$

$\int xe^{xy}dy = e^{xy} + h(x)$

$f(x,y) = e^{xy} + C$

$(a)\mathbf{r}_1(t) = t\mathbf{i} - (t-3)\mathbf{j},\ \ 0 \le t \le 3$

$$\int_C \mathbf{F}\cdot d\mathbf{r} = [e^{xy}]_{(0,3)}^{(3,0)} = e^{(3)(0)} - e^{(0)(3)} = 1 - 1 = \boxed{0}$$

(b)Since the interval is closed and the vector field is conservative, it follows that

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \boxed{0}$$

2. (MAPLE) Section 14.3, Page 1039, question 30.

$\int_C \frac{2x}{(x^2+y^2)^2}dx + \frac{2y}{(x^2+y^2)^2}dy$

$\int \frac{2x}{(x^2+y^2)^2}dx = -\frac{1}{x^2+y^2} + g(y)$

$\int \frac{2y}{(x^2+y^2)^2}dy = -\frac{1}{x^2+y^2} + h(x)$

$f(x,y) = -\frac{1}{x^2+y^2} + C$

$$\int_C \frac{2x}{(x^2+y^2)^2}dx + \frac{2y}{(x^2+y^2)^2}dy = [-\frac{1}{x^2+y^2}]_{(7,5)}^{(1,5)} = -\frac{1}{1^2+5^2} + \frac{1}{7^2+5^2} = \frac{1}{74} - \frac{1}{26} = \frac{26-74}{74(26)} = \frac{-48}{74(26)} = \frac{-12}{37(13)} = \boxed{\frac{-12}{481}}$$

Maple Verification:

```
> with(VectorCalculus):
> SetCoordinates( cartesian[x,y] );
```

$$cartesian_{x,y}$$

```
> LineInt( VectorField( <2*x/(x^2+y^2)^2,2*y/(x^2+y^2)^2> ), Arc( Circle( <4,5>, 3 ), 0, -Pi ) );
```

$$\frac{-12}{481}$$

3. Section 14.4, Page 1048, question 10.

$\int_C (y-x)dx + (2x-y)dy$

$\therefore M = y - x \Rightarrow \frac{\partial M}{\partial y} = 1 \quad N = 2x - y \Rightarrow \frac{\partial N}{\partial x} = 2 \therefore \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 2 - 1 = 1$

$$\int_C (y-x)dx + (2x-y)dy = \int_0^\pi \int_3^5 1 r dr d\theta = \int_0^\pi \int_3^5 1 r dr d\theta = \frac{1}{2}\int_0^\pi [r^2]_3^5 d\theta = \frac{1}{2}\int_0^\pi 16 d\theta = \frac{1}{2}[16\theta]_0^\pi = \boxed{8\pi}$$

4. (MAPLE) Section 14.4, Page 1049, question 34.

$A = \int_{-a}^{a}\sqrt{a^2 - x^2}dx = \frac{\sqrt{a^2}\pi}{2\sqrt{\frac{1}{a^2}}} = \frac{\pi a^2}{2} \quad C_1 : y = \sqrt{a^2 - x^2}, \quad C_2 : y = 0$

$\bar{x} = \frac{1}{2A}\int_{C_1} x^2 dy + \frac{1}{2A}\int_{C_2} x^2 dy \qquad For C_1 : dy = -\frac{x}{\sqrt{a^2-x^2}}, For C_2 : dy = 0 \therefore \bar{x} = \frac{1}{2A}\int_{-a}^{a} x^2(-\frac{x}{\sqrt{a^2-x^2}})dx =$

$\bar{y} = \frac{1}{2A}\int_{C_1} y^2 dx + \frac{1}{2A}\int_{C_2} y^2 dx \qquad Since\ along\ \ C_2, y = 0 \therefore \bar{y} = \frac{1}{2A}\int_{a}^{-a}(\sqrt{a^2 - x^2})^2 dx = \frac{4\sqrt{\frac{1}{2}a^2}^{\,3}}{3\sqrt{a^2}\pi} = \frac{4a}{3\pi}$

$(\bar{x}, \bar{y}) = \boxed{(0, \frac{4a}{3\pi})}$