Part I

1. Page 866, 51.

$(a) w=\sqrt{x^2+y^2+z^2}=(x^2+y^2+z^2)^{1/2}$

$\frac{\partial w}{\partial x}=\frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2x)=\frac{2x}{2\sqrt{x^2+y^2+z^2}}=\boxed{\frac{x}{\sqrt{x^2+y^2+z^2}}}$

$\frac{\partial w}{\partial y}=\frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2y)=\frac{2y}{2\sqrt{x^2+y^2+z^2}}=\boxed{\frac{y}{\sqrt{x^2+y^2+z^2}}}$

$\frac{\partial w}{\partial z}=\frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2z)=\frac{2z}{2\sqrt{x^2+y^2+z^2}}=\boxed{\frac{z}{\sqrt{x^2+y^2+z^2}}}$

$(b)\frac{\partial^2 w}{\partial x^2}=\frac{\sqrt{x^2+y^2+z^2}-x\frac{x}{\sqrt{x^2+y^2+z^2}}}{(\sqrt{x^2+y^2+z^2})^2}=\frac{\sqrt{x^2+y^2+z^2}-\frac{x^2}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}(\frac{\sqrt{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}})=\frac{x^2+y^2+z^2-x^2}{x^2+y^2+z^2\sqrt{x^2+y^2+z^2}}=\frac{y^2+z^2}{x^2+y^2+z^2\sqrt{x^2+y^2+z^2}}$

$\frac{\partial^2 w}{\partial y^2}=\frac{\sqrt{x^2+y^2+z^2}-y\frac{y}{\sqrt{x^2+y^2+z^2}}}{(\sqrt{x^2+y^2+z^2})^2}=\frac{\sqrt{x^2+y^2+z^2}-\frac{y^2}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}(\frac{\sqrt{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}})=\frac{x^2+y^2+z^2-y^2}{x^2+y^2+z^2\sqrt{x^2+y^2+z^2}}=\frac{x^2+z^2}{x^2+y^2+z^2\sqrt{x^2+y^2+z^2}}$

$\frac{\partial^2 w}{\partial z^2}=\frac{\sqrt{x^2+y^2+z^2}-z\frac{z}{\sqrt{x^2+y^2+z^2}}}{(\sqrt{x^2+y^2+z^2})^2}=\frac{\sqrt{x^2+y^2+z^2}-\frac{z^2}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}(\frac{\sqrt{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}})=\frac{x^2+y^2+z^2-z^2}{x^2+y^2+z^2\sqrt{x^2+y^2+z^2}}=\frac{x^2+y^2}{x^2+y^2+z^2\sqrt{x^2+y^2+z^2}}$

$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}=\frac{y^2+z^2}{x^2+y^2+z^2\sqrt{x^2+y^2+z^2}}+\frac{x^2+z^2}{x^2+y^2+z^2\sqrt{x^2+y^2+z^2}}+\frac{x^2+y^2}{x^2+y^2+z^2\sqrt{x^2+y^2+z^2}}=\frac{2(x^2+y^2+z^2)}{x^2+y^2+z^2\sqrt{x^2+y^2+z^2}}=\boxed{\frac{2}{\sqrt{x^2+y^2+z^2}}}$

2. Page 882, 31.

$x^2+y^2+z^2=25 \Rightarrow x^2+y^2+z^2-25=0$

$F_x(x,y,z)=2x$

$F_y(x,y,z)=2y$

$F_z(x,y,z)=2z$

$\frac{\partial z}{\partial x}=-\frac{F_x(x,y,z)}{F_z(x,y,z)}=\boxed{-\frac{x}{z}}$

$\frac{\partial z}{dy}=-\frac{F_y(x,y,z)}{F_z(x,y,z)}=\boxed{-\frac{y}{z}}$

3. Page 893, 27.

$g(x,y)=x^2+y^2+1$

$\frac{\partial g}{\partial x}=2x, \qquad \frac{\partial g}{\partial y}=2y$

$\nabla g(x,y)=\frac{\partial g}{\partial x}\mathbf{i}+\frac{\partial g}{\partial y}\mathbf{j}=(2x)\mathbf{i}+(2y)\mathbf{j}$

$\nabla g(1,2)=(2(1))\mathbf{i}+(2(2))\mathbf{j}=2\mathbf{i}+4\mathbf{j}$

$P(1,2), Q(3,6) \Rightarrow \overrightarrow{\mathbf{PQ}}=(3-1)\mathbf{i}+(6-2)\mathbf{j}=2\mathbf{i}+4\mathbf{j} \Rightarrow \|\overrightarrow{\mathbf{PQ}}\|=\sqrt{2^2+4^2}=\sqrt{20}$

$\mathbf{u}=\frac{2}{\sqrt{20}}\mathbf{i}+\frac{4}{\sqrt{20}}\mathbf{j}$

$D_u f(1,2)=\nabla g(1,2)\cdot\mathbf{u}=(2\mathbf{i}+4\mathbf{j})\cdot(\frac{2}{\sqrt{20}}\mathbf{i}+\frac{4}{\sqrt{20}}\mathbf{j})=\frac{4}{\sqrt{20}}+\frac{16}{\sqrt{20}}=\frac{20}{2\sqrt{5}}=\frac{10}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}}=\frac{10\sqrt{5}}{5}=\boxed{2\sqrt{5}}$

4. Page 903, 31.

$xy - z = 0$

$F_x = y, \qquad F_y = x, \qquad F_z = -1$

$\nabla F(x,y,z) = y\mathbf{i} + x\mathbf{j} - \mathbf{k}$

$\nabla F(-2,-3,6) = -3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$

$TangentPlane: F_x(x_0,y_0,z_0)(x - x_0) + F_y(x_0,y_0,z_0)(y - y_0) + F_z(x_0,y_0,z_0)(z - z_0) = 0$

$-3(x+2) - 2(y+3) - (z-6) = 0$

$-3x - 6 - 2y - 6 - z + 6 = 0$

$\boxed{-3x - 2y - z - 6 = 0}\ OR\ \boxed{3x + 2y + z = -6}$

$NormalLine:$

$\boxed{\frac{x+2}{-3} = \frac{y+3}{-2} = \frac{z-6}{-1}}\ OR\ \boxed{\frac{x+2}{3} = \frac{y+3}{2} = \frac{z-6}{1}}$

Part II.

5. Page 866, 84.

$Let: z = f(x,y)$

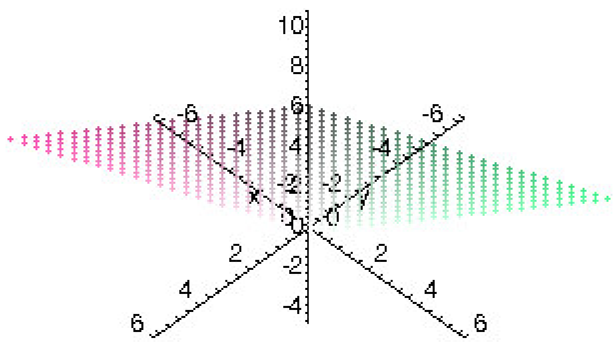
$If: f(x,y) = -\frac{3}{4}x + \frac{1}{2}y + 3 \qquad Then: \qquad f_x = -\frac{3}{4}, f_y = \frac{1}{2}$

$z = -\frac{3}{4}x + \frac{1}{2}y + 3 \Rightarrow 4z = -3x + 2y + 12 \Rightarrow 3x - 2y + 4z = 12$

$xyTrace - Let: z = 0 \Rightarrow 3x - 2y = 12$

$yzTrace - Let: x = 0 \Rightarrow 4z - 2y = 12$

$xzTrace - Let: y = 0 \Rightarrow 3x + 4z = 12$



6. Page 883, 52.

$Given: \frac{\partial r}{\partial t} = 6in/\min, \qquad \frac{\partial h}{\partial t} = -4in/\min$

$V = \pi r^2 h$

$\frac{\partial V}{\partial t} = \pi(r^2 \frac{\partial h}{\partial t} + 2rh\frac{\partial r}{\partial t}) = \pi(r^2(-4) + 2rh(6)) = \pi(12rh - 4r^2)$

$\frac{\partial V}{\partial t}|_{r=12,h=36} = \pi(12(12)(36) - 4(12)^2) = \pi(144(36) - 4(144)) = 144\pi(32) = \boxed{4608\pi \frac{in^3}{\min}}$

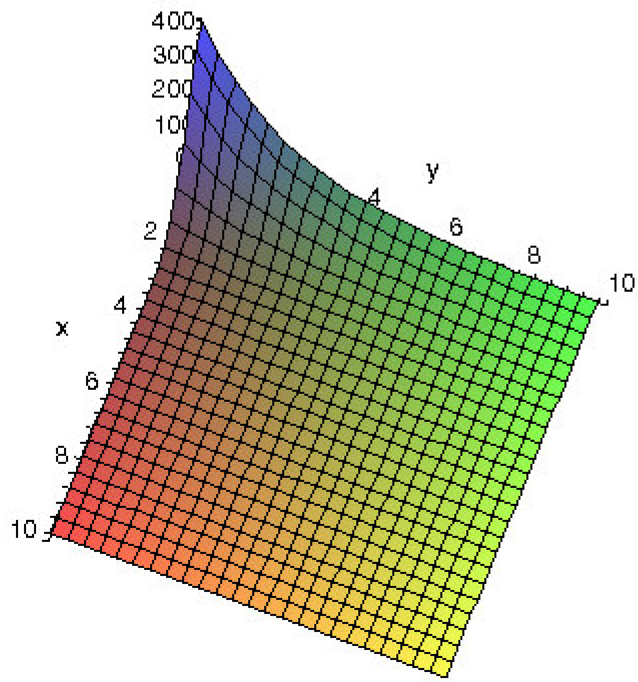
$S = 2\pi rh$

$\frac{\partial S}{\partial t} = 2\pi(r\frac{\partial h}{\partial t} + h\frac{\partial r}{\partial t}) = 2\pi(r(-4) + h(6)) = 2\pi(6h - 4r)$

$\frac{\partial S}{\partial t}|_{r=12,h=36} = 2\pi(6(36) - 4(12)) = 24\pi(18 - 4) = (14)24\pi = \boxed{336\pi \frac{in^2}{\min}}$

7. Page 895, 76.

(a)



(b)

$T(x,y) = 400e^{-(x^2+y)/2} = 400e^{(-x^2/2)+(-y/2)}$

$\frac{\partial T}{\partial x} = -400xe^{(-x^2/2)+(-y/2)}, \qquad \frac{\partial T}{\partial y} = -200e^{(-x^2/2)+(-y/2)}$

$\nabla T(x,y) = \frac{\partial T}{\partial x}\mathbf{i} + \frac{\partial T}{\partial y}\mathbf{j} = (-400xe^{(-x^2/2)+(-y/2)})\mathbf{i} - (200e^{(-x^2/2)+(-y/2)})\mathbf{j}$

$\nabla T(3,5) = (-400(3)e^{(-3^2/2)+(-5/2)})\mathbf{i} - (200e^{(-3^2/2)+(-5/2)})\mathbf{j} = (-1200e^{-14/2})\mathbf{i} - (200e^{-14/2})\mathbf{j} = (-1200e^{-7})\mathbf{i} - (200e^{-7})\mathbf{j}$

$NoChangeInT \therefore \nabla T(3,5)\cdot\mathbf{b} = 0$

$\boxed{\mathbf{b} = \frac{1}{\sqrt{37}}\mathbf{i} - \frac{6}{\sqrt{37}}\mathbf{j}}$ *AND* $\boxed{\mathbf{b} = -\frac{1}{\sqrt{37}}\mathbf{i} + \frac{6}{\sqrt{37}}\mathbf{j}}$

(c)

$Maximize: \|\nabla T(3,5)\| * \|\cdot\mathbf{u}\| * \cos\theta \therefore \theta = 0$

$\boxed{\mathbf{u} = \frac{-1200e^{-7}}{\sqrt{1480000e^{-14}}}\mathbf{i} - \frac{200e^{-7}}{\sqrt{1480000e^{-14}}}\mathbf{j}}$ *OR* $\boxed{\mathbf{u} = -\frac{6}{\sqrt{37}}\mathbf{i} - \frac{1}{\sqrt{37}}\mathbf{j}}$

8. Page 902, 28.

$x = y(2z-3) \Rightarrow x - 2yz + 3y = 0$

$F_x(x,y,z) = 1, \qquad F_y(x,y,z) = 3 - 2z, \qquad F_z(x,y,z) = -2y$

$F_x(4,4,2) = 1, \qquad F_y(4,4,2) = 3 - 2(2) = -1, \qquad F_z(4,4,2) = -2(4) = -8$

$TangentPlane: F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0) = 0$

$1(x-4) - 1(y-4) - 8(z-2) = 0$

$x - 4 - y + 4 - 8z + 16 = 0$

$\boxed{x - y - 8z = -16}$