Diff. Eq.

Complex roots

Example:

\[ y'' + y' + y = 0 \]

Guess:

\[ y = e^{rt} \]
\[ y' = rer^t \]
\[ y'' = r^2er^t \]

\Rightarrow \text{characteristic equation is}

\[ r^2 + r + 1 = 0 \]
Diff Eq.

2 linearly independent real solutions

\[ y_1 = e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right), \quad y_2 = e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right) \]

Initial conditions

\[ y(0) = 1 \]
\[ y'(0) = 2 \]

\[ y = c_1 y_1 + c_2 y_2 \]

Find \( c_1, c_2 \)
Graph the solution.
Diff Eq.

Principle of Superposition:
If \( y_1, y_2 \) are solutions of
\[
y'' + p(t)y' + q(t)y = 0
\]
then \( c_1 y_1, c_2 y_2 \) are solutions
and \( c_1 y_1 + c_2 y_2 \) is a solution
\[c_1, c_2 \text{ are constants}\]

Wronskian Determinant:
Let \( y_1, y_2 \) be solutions of
\[
y'' + p(t)y' + q(t)y = 0
\]
Then the Wronskian determinant is
\[
W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \text{constant}
\]
If \( y_1, y_2 \) are linearly independent (l.i.), then

\[ W \neq 0. \]

For any value of \( t \).

Example: \( y'' + 9y = 0 \)

\[ y_1 = \sin 3t \quad y_2 = \cos 3t \]

If \( y_1, y_2 \) are linearly dependent then \( W = 0 \).
For the equation
\[ y'' + p(t)y' + q(t)y = 0 \]
For any two solutions \( y_1, y_2 \)
\[ W = c \exp \left[ -\int p(t) \, dt \right] \]
The reason is that
\[ W = y_1 y_2' - y_1' y_2 \]
satisfies
\[ W' + p(t)W = 0. \]