Escape Velocity

Gravitational force \( w(x) = \frac{-k}{(x+R)^2} \)

\( k = \text{constant} \)

\( w(0) = -mg \quad \Rightarrow \quad w(x) = \)

Assume no drag due to atmosphere

\( \Rightarrow \quad \frac{mdv}{dt} = \)

with \( v(0) = v_0, \quad \) some \( v_0 \)

Change independent variables from \( t \) to \( x \)
Then equation becomes

$$\frac{vdv}{dx} =$$

$$v(0) = v_0$$

Solve using separable equations

Find maximum altitude
Find initial velocity that makes maximum altitude \( \to \infty \) (this initial velocity is called the escape velocity).

Radio carbon dating:

Let \( Q(t) \) be the amount of carbon-14 at time \( t \).

Suppose \( Q_0 \) is the amount of carbon-14 at time \( t=0 \).

Suppose \( Q \) satisfies

\[
\frac{dQ}{dt} = -k Q
\]

Suppose it is known that after 50,000 years \( \frac{Q(t)}{Q_0} = 0.00236 \). Find an expression for \( t \).
Suppose some remains are discovered where the residual amount of carbon-14 is 20%. Find the age of these remains.

Differences between linear and non-linear equations:

Linear: If \( p(t) \) and \( g(t) \) are continuous on \( \alpha < t < \beta \) and \( \alpha < t_0 < \beta \) then there is exactly one solution of

\[
y' + p(t)y = g(t)\]

\[
y(t_0) = y_0
\]

in the whole interval \( \alpha < t < \beta \).
The formula for the solution is:

Nonlinear Equations:

\[ y' = f(t, y) \quad \alpha < t < \beta, \quad \alpha < y < \beta \]
\[ y(t_0) = y_0 \quad \alpha < t_0 < \beta \]

1. Solution may not exist in whole interval
   Example:
2. There may be two solutions:
   Example:

3. Solution always exists in small interval $t_0 - h < t < t_0 + h$ when $f$ and $\frac{df}{dy}$ are continuous.

Note: This means solutions never cross.