Integrating factors

Example 1:

Solve: \[ \frac{dy}{dt} + 2y = 3 \]

Recall \[ \frac{dv}{dt} = \frac{9.8 - \frac{1}{5}v}{v - 45} = -\frac{1}{5} \]

\[ \frac{d}{dt} \ln |v - 45| = -\frac{1}{5} \]

\[ \frac{d}{dt} e^{2t} = 2e^{2t} \]

Technique: Multiply equation by 'something' so the left hand side is the derivative of a product.
Add initial condition
\[ y(0) = 1 \]
\[ y(0) = 3 \]

Example 2:
\[(t+1) \frac{dy}{dt} + 2y = 3\]
Suggestion:
\[ \frac{d}{dt} (t+1)^2 = 2(t+1) \]
Multiply equation by 'something' so left hand side is the derivative of a product.
Add initial condition

\[ y(10) = 1 \]

Method:

\[ \frac{dy}{dt} + 5y = 6 \]

Multiply by \( u(t) \)

\[ u(t)y' + 5uy = 6u \]

Want

\[ 5u = u' = \frac{du}{dt} \]

Because then

\[ u'y + u'y = \frac{d}{dt} (uy) = 6u \]

Solve

\[ \frac{du}{dt} = 5u \]
We don't add an initial condition for $u$. Why?

Another Example:

$$(t+1)^2 y' + y(t+1) y = 5$$
\[ \frac{dy}{dt} + p(t)y = g(t) \]

\[ \mu \frac{dy}{dt} + \mu p(t)y = g(t)\mu(t) \]

\[ \mu' = \mu p(t) \]