Consider mass-spring system

1 lb mass extends a spring 6 in.

There is a viscous force of \(-2\) lbs for a velocity of 1 ft/sec.

Show the equation for the displacement, \(u(t)\), is

\[
\frac{1}{32} u'' + 2u' + 2u = 0
\]

We can write the second order equation as a first order system.

Method: let \(u' = v\)

Show system of equations is

\[
\begin{align*}
    u' &= v \\
    v' &= -64u - 64v
\end{align*}
\]
Rewrite equation using matrices and vectors
\[ \mathbf{y}(t) = \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \]
\[ \mathbf{y}'(t) = \begin{pmatrix} u'(t) \\ v'(t) \end{pmatrix} = \begin{pmatrix} \mathbf{a}(t) \\ \mathbf{b}(t) \end{pmatrix} \]

A solution to this equation is a vector \( \mathbf{u}(t) \)

Write the equation
\[ u'' + 3u' + 2u = 0 \]
as a first order system
Recall facts about matrices: let $A$ be a $2 \times 2$ matrix.

$A$ is nonsingular or noninvertible iff

$A$ is singular or noninvertible iff

Remember if $A$ and $B$ are $2 \times 2$ matrices

then usually

$AB \neq BA$
Examples:

1. \( A_1 = \begin{pmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 1 \end{pmatrix} \) singler or nonsingular?

2. \( A_2 = \begin{pmatrix} 1 & 1 \\ \frac{1}{2} & 1 \end{pmatrix} \) singler or nonsingular

3. \( A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \). Show \( A^{-1} = \frac{\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}{\det A} \)

   if \( \det A \neq 0 \)

5. Find \( A_2^{-1} \).
5. Find nonzero $x$ such that $Ax = 0$

Graph all possible solutions.

Eigenvalues of Matrices

Let $I = (1, 0)$.

**Def**: $\lambda$ is an eigenvalue for $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

iff $A - \lambda I = \begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix}$ is singular.

Return to examples:

1. Find the eigenvalues of $A$. 

5. Find nonzero $x$ such that $Ax = 0$.
2. For each eigenvalue, \( \lambda_1 \) and \( \lambda_2 \), find the nonzero solutions of

\[
(A - \lambda I)x = 0
\]

\[
(A - \lambda_2 I)x = 0
\]
3. Find the eigenvalues of $A_2$.

4. For each eigenvalue, $\lambda_1$ and $\lambda_2$, of $A_2$, find the non-zero solutions of

$$(A_2 - \lambda I)x = 0$$
$$(A_2 - \lambda_2 I)x = 0$$