We use \[ \sum_{n=1}^{\infty} \cos \frac{n\pi}{L} \frac{x}{L} \quad \sum_{n=1}^{\infty} \sin \frac{n\pi}{L} \frac{x}{L} \]
to expand functions defined on \(-L \leq x \leq L\).

Where do these functions come from?

First we make an aside: Recall what we mean by an eigenvalue of a matrix:
Now consider an eigenvalue problem for a differential equation.

Two parts:
- differential equation $y'' + u^2 y = 0$
- boundary conditions $y(0) = y(\pi) = 0$

Goal:

Now do we find the eigenvalues and eigenfunctions:
Any solution of the diff. eq. \( y'' + \omega^2 y = 0 \) has the form
\[ y(x) = c_1 \sin \omega x + c_2 \cos \omega x \]

Impose the boundary condition \( y(0) = 0 \)

Impose the boundary condition \( y(\pi) = 0 \)
Second Eigenvalue Problem:

\[ y'' + \lambda^2 y = 0 \quad 0 < x < \pi \]

\[ y(0) = 0 = y'(\pi) \]
Second Eigenvalue Problem (cont.)