Diff. Eq.

Return to Example (p. 67, Worksheets)

Mass weighing 8 pounds stretches spring 1.5 inches. Now suppose also that there is an external oscillating force 3 cos 14t pounds.

\[ m = \frac{8 \text{ lb}}{32 \text{ ft/s}^2} \quad \text{mg} = 8 = k \cdot \frac{1}{8} \]

\[ \frac{1}{4} u'' + 64 u = 3 \cos 14t \]

Two linearly independent solutions of

\[ \frac{1}{4} u'' + 64 u = 0 \]

are

\[ u_1 = \sin 16t \quad u_2 = \cos 16t \]

For the nonhomogeneous part

\[ U = A \cos 14t + B \sin 14t \]

\[ U = \frac{1}{5} \cos 14t \]

so the general solution is

\[ c_1 \sin 16t + c_2 \cos 16t + \frac{1}{5} \cos 14t \]
Suppose the mass starts at rest.

\[ u(0) = 0 \]
\[ u'(0) = 0 \]

Solution is

\[ u(t) = \frac{1}{5} \left[ \cos 14t - \cos 16t \right] \]

\[ = \frac{2}{5} \sin t \sin 15t \]

Beats.
Resonance:

Now suppose the external force is \(3 \cos 16t\).
Now this is a solution of the homogeneous equation.
In this case, for the solution to the non-homogeneous eq.
Guess:

\[ u = At \sin 16t \]

General solution is:

\[ c_1 \sin 16t + c_2 \cos 16t + \frac{3}{8} t \sin 16t \]

let \( z(0) = 0 \) and \( z'(0) = 0 \). What is the solution?
Freed Vibrations with damping:

Start with our model with damping

\[ \frac{1}{\gamma} u'' + 2u' + 64u = 3 \cos 14t \]

and the external oscillating force \( 3 \cos 14t \) pounds

(damping force is \(-6 \) pounds when the velocity is \( 3 \text{ ft/sec} \).)
Damped Vibrations with Damping

The solutions to the homogeneous equations are

\[ u_1 = e^{-4t} \cos 15t, \quad u_2 = e^{-4t} \sin 15t \]

Using the method of undetermined coefficients, a solution of the nonhomogeneous equation is

\[ U = A \cos 14t + B \sin 14t \]

\[ U = 0.174 \cos 14t + 0.325 \sin 14t \]

General solution is

\[ u = c_1 e^{-4t} \cos 15t + c_2 e^{-4t} \sin 15t + 0.174 \cos 14t + 0.325 \sin 14t \]
When there are forced vibrations and the system has damping, what is the long time behavior? We call that part of the solution the steady part.