Example:

\[ y'' + 9y = e^{3t} \sin 2t + t^2 \]

I. Find particular solution of non-homogeneous equation

Guess: \[ Y = Ae^{3t} \sin 2t + Be^{3t} \cos 2t + Ct^2 + Dt + E \]
Solutions of homogeneous equation are \( y_1, y_2 \). Give 2 k.i. solutions for the homogeneous equation.

The general solution is.
Diff. Eq.

The variation of parameters formula:

Example: Consider

\[ y'' + 9y = e^{(t^2 \cos t)} = g(t) \]

Two linearly independent solutions of \( y'' + 9y = 0 \) are \( y_1 = \cos 3t, \ y_2 = \sin 3t \).

The Wronskian of \( y_1, y_2 \) is

\[
W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1',
\]

\[
= \frac{y_1 y_2'}{2} - \frac{y_1' y_2}{2}
\]

\[
= 3
\]
Diff Eq.

The solution of the nonhomogeneous equation gives the variation of parameters formula

\[ y(t) = -y_1(t) \int_{t_0}^{t} \frac{y_2(s) g(s) ds}{W(y_1, y_2)} + y_2(t) \int_{t_0}^{t} \frac{y_1(s) g(s) ds}{W(y_1, y_2)} \]
Calculate

$Y' = \,\,$

$Y'' = \,\,$

Find $Y'' + 9Y$

Excellent Guess! General solution is:
Diff. Eq.

This method always works!

Write the general solution for

\[ y'' - 7y' + 10y = e^{5t^2} \]

using the variation of parameters formula.