What are Differential Equations?

1. Algebraic Equations: Examples

\[ \dot{x} + 5 = 0 \]

Solutions are a collection of numbers.

2. Differential Equations: Examples

\( (a) \quad \frac{dv}{dt} = 9.8 - \frac{1}{2} v \)

What is a solution of this equation?

Solution is a function \( v = v(t) \).

The goal is to find an expression for this function.
Differential Equations: Examples (cont.)

(b) \[ \frac{dp}{dt} = 0.5p - 450 \]

What is the solution of this equation?

How do differential equations arise?
They describe physical, biological, chemical, economic, financial or other processes.

Here's how example (a) is derived:
Mass falling in the atmosphere

\[ F_{\text{force due to gravity}} - F_{\text{force due to drag}} = ma \]

Let \( v \) be the velocity of the mass.
Start with Newton's law of motion:

\[ F = ma \]
\[ ma = \frac{m \, dv}{dt} = mg - \alpha v \]

**Example:**

- \( m = 10 \, \text{kg} \)
- \( g = 9.8 \, \text{meters/(sec)}^2 \)
- \( \alpha = 2 \, \text{deg/(sec)}^2 \)

\[
\frac{dv}{dt} = 9.8 - \frac{1}{5} v = \frac{49 - v}{5}
\]

\[ v(0) = 0 \]
Example:
\[
\frac{dv}{dt} = 9.8 - \frac{1}{2} v
\]
\[v(0) = 0\]

Find the solution to this problem.