This is a **CLOSED BOOK** test.

1. Please answer all questions, showing your work in detail and giving reasons where appropriate.

2. Collaboration with other students is **NOT** permitted.

3. Be sure you have 6 test pages for this test.

4. Point allocations for each question are indicated. Plan your time accordingly. The total number of points is 100.

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1. Suppose a mass weighing 8 pounds extends a spring 24 inches. Suppose there is an external force of \((1/2) \cos 4t\) pounds. Suppose also the mass is initially at rest.

(a) (8 points) Write the differential equation and initial conditions for this problem, (using feet as the length units);

(b) (12 points) Find the solution for the problem in part (a); [Hint: For the solution of the nonhomogenous problem guess \(U(t) = At \sin 4t\).]
(c) (10 points) Graph your solution using the correct initial conditions. What happens to the solution as $t \to \infty$. 
2. Let \( f : [-2, 2) \rightarrow \mathbb{R} \)

\[
f(x) = \begin{cases} 
-1, & -2 \leq x < -1 \\
1, & -1 \leq x < 0 \\
-1, & 0 \leq x < 1 \\
1, & 1 \leq x < 2 
\end{cases}
\]

(a) (12 points) Is \( f(x) \) on \([-2, 2]\) even, odd, or neither? Extend \( f(x) \) periodically to \([-4, 4]\). Plot \( f(x) \) on \([-4, 4]\).

(b) (15 points) Find the corresponding Fourier series for \( f \). If \( f(x) \) is even or odd, take advantage of this in finding the Fourier coefficients \( a_m \) and \( b_m \).
(c) (8 points) Sketch the graph of the function to which the Fourier series converges on $[-2, 2]$.

3. (a) (20 points) Use separation of variables to split the following partial differential equation and boundary conditions into one ordinary differential equation and one eigenvalue problem. (Please proceed in a logical manner and include all of the standard arguments)

$$4u_{xx} = u_{tt}, \quad 0 < x < L, \quad t > 0,$$

$$u(0, t) = 0, \quad u(L, t) = 0, \quad 0 < x < L.$$
(b) (15 points) Given the following general solution and initial condition to a particular heat conduction problem, find the sequence of \( c_n \)'s which give the specific solution

\[
\begin{align*}
    u_{\text{gen}}(x,t) &= \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n e^{-\frac{n^2 \pi^2 t}{16}} \cos\left(\frac{n\pi x}{4}\right), \quad 0 < x < 4, \quad t > 0, \\
    u(x,0) &= x, \quad 0 < x < 4.
\end{align*}
\]