This is a CLOSED BOOK test.

1. Please answer all questions, showing your work in detail and giving reasons where appropriate.

2. Collaboration with other students is NOT permitted.

3. Be sure you have 6 test pages for this test.

4. Point allocations for each question are indicated. Plan your time accordingly. The total number of points is 100.

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1. Solve the following equation by finding an explicit form of the solution \( y(x) \) in:

(a) (10 points)

\[
\frac{d}{dx} y = -\frac{yx}{x^2 + 1}, \quad y(0) = 1.
\]

(b) (10 points)

\[
\frac{d}{dt} y = (t + 1)y, \quad y(0) = 3.
\]
2. A certain valley contains a population of 3-toed sloths which is modeled by the following logistic equation.

\[
\frac{dS}{dt} = -\frac{1}{2} \left( 1 - \frac{S}{200} \right) \left( 1 - \frac{S}{630} \right)
\]

(a) (10 points) Draw a stability diagram \( \frac{dS}{dt} vs S \) for this autonomous differential equation.

\[
\begin{array}{c}
\frac{dS}{dt} \\
\hline
S
\end{array}
\]

(b) (10 points) Determine all equilibrium solutions.
(c) (10 points) If this population started with 250 sloths describe the evolution of this population (i.e. what would happen as $t \to \infty$.)

3. (a) (10 points) Find two linearly independent solutions, $y_1, y_2$, of $y'' + 6y' + 8y = 0$;
(b) (10 points) Using your answer from part (a) find the solution of
\[y'' + 6y' + 8y = 0, \quad y(0) = 3, \quad y'(0) = 1;\]

(c) (10 points) Graph the solution you obtain in part (b), showing the
behavior as \( t = 0 \) and as \( t \to \infty \).
4. The two linearly independent solutions of $y'' + 25y = 0$ are $\sin 5t$ and $\cos 5t$.

(a)  (10 points) Write down the variation of parameter formula for a solution of $y'' + 25y = e^{at}$, evaluating the Wronskian in this formula.

(b)  (10 points) Use your variation of parameters formula to write down the general solution of $y'' + 25y = e^{at}$. 