This is a CLOSED BOOK test.

1. Please answer all questions, showing your work in detail and giving reasons where appropriate.

2. Collaboration with other students is NOT permitted.

3. Be sure you have 6 test pages for this test.

4. Point allocations for each question are indicated. Plan your time accordingly. The total number of points is 100.

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1. A tank contains 200 liters of fluid in which 60 grams of salt is dissolved. A brine solution of 2 grams of salt per liter is pumped into the tank at a rate of 2 liters/minute. The well-stirred solution is then pumped out at the same rate.
(a) (10 points) Write the differential equation in terms of $Q(t)$ and $t$ and write the initial condition to model this problem. ($Q(t)$ should represent the amount of salt in the tank at time $t$).

(b) (10 points) Solve the equation in part (a).
1. (c) (10 points) How much salt will be present in the tank after a long period of time?

2. A certain valley contains a population of 3-toed sloths which is modeled by the following logistic equation.

\[
\frac{dS}{dt} = -\frac{1}{5} \left( 1 - \frac{S}{100} \right) \left( 1 - \frac{S}{565} \right) S
\]

(a) (10 points) Draw a stability diagram \(\frac{dS}{dt} vs S\) for this autonomous differential equation.
2. (b) (10 points) Determine all equilibrium solutions.

(c) (10 points) If this population started with 236 sloths describe the evolution of this population (i.e. what would happen as $t \to \infty$)
3. (a) (10 points) Find two linearly independent solutions, $y_1, y_2,$
of $y'' + 16y = 0.$

(b) (10 points) Using your answer from part (a) find the
solution of $y'' + 16y = 0, y(0) = 2, y'(0) = 1.$
3. (c) (10 points) Find the Wronskian of the two linearly independent solutions you found in part (a).

4. (a) (10 points) Find the general solution of \( y'' - 6y' + 8y = sint \).