Hwk 4 math 4740 due for in class mock exam Oct 16 2 - 3 pm

1. Consider the 3 by 2 single period three states market model with interest rate $r = 1/9$

$$M = \begin{bmatrix} 1 + 1/9 & 20/3 \\ 1 + 1/9 & 4/5 \\ 1 + 1/9 & 10/3 \end{bmatrix}$$

Compute rank( $M$).

2. Given above $M$, compute a replicating vector $(b, s)^t$ for the payoff vector $V_1 = (1, 0, -1/2)^t$ of a forward contract like derivative.

3. Find a second contingency claim $V_2 = (X_u^1, X_m^1, X_d^1)^t$, different from that in Q2 that can be replicated in the same market $M$.

4. It turns out there are infinite number of distinct claims $V$ that can be replicated in the market $M$. Give conditions on the components of $V = (X_u^1, X_m^1, X_d^1)^t$ that completely specify this set of claims in the market $M$.

5. Since $M$ is incomplete, and you are given the additional fact that $M$ has no Arbitrage with $S_0 = 5$, you should expect to find many distinct Risk Neutral probability measure $Q = (Q_u > 0, Q_m > 0, Q_d > 0)^t$ such that $M'Q = \frac{10}{9}(1, 5)^t$. Compute one risk neutral $Q$ for this market.

6. There are actually infinite number of distinct risk neutral $Q$ for this market. Compute all of them analytically and show directly that for each of the replicable claims in this market, the risk neutral prices under different $Q$ and $Q'$ agree, that is, for each replicable $V = (X_u^1, X_m^1, X_d^1)^t$ under $M$,

$$E_Q[9V/10] = E_{Q'}[9V/10].$$