

Locally Divergence-Free Discontinuous Galerkin Methods for MHD Equations

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Abstract

In this paper, we continue our investigation of the locally divergence-free discontinuous Galerkin method, originally developed for the linear Maxwell equations jointly with Cockburn, to solve the nonlinear ideal magnetohydrodynamics (MHD) equations. The distinctive feature of such methods is the use of approximate solutions that are exactly divergence-free inside each element for the magnetic field. As a consequence, this method has a smaller computational cost than the traditional discontinuous Galerkin method with standard piecewise polynomial spaces. We formulate the divergence-free discontinuous Galerkin method for the MHD equations and perform extensive one and two dimensional numerical experiments for both smooth solutions and solutions with discontinuities. Our computational results demonstrate that the divergence-free discontinuous Galerkin method, with a reduced cost comparing to the traditional discontinuous Galerkin method, can maintain the same accuracy for smooth solutions and can enhance the numerical stability of the scheme and reduce certain nonphysical features in some of the test cases.