HW1: MATH/CSCI-4800-02 Numerical Computing
Due 4pm on 1.22.2019 (Tuesday)

Note: Page numbers below are with respect to the textbook in 3rd edition. The actual problem descriptions are also given below, in case you are using a different edition of the textbook.

1. Text exercises on page 5: 6.(b)
   Explain how to evaluate the polynomial for a given point $x$, using as few operations as possible. How many multiplications and how many additions are required?
   
   \[ P(x) = a_7 x^7 + a_{12} x^{12} + a_{17} x^{17} + a_{22} x^{22} + a_{27} x^{27} \]

2. Computer problem on page 5: 2 (either use nest.m or myPolyEval.m shared by Prof. Li).
   Evaluate \( P(x) = 1 - x + x^2 - x^3 + \ldots + x^{98} - x^{99} \) at \( x = 1.00001 \). Find a simpler, equivalent expression, and use it to estimate the error of the nested multiplication.

3. Convert the following binary numbers to base 10: (a) 101.101, (b) 10.101

4. Text exercises on page 16: 6.(a) (check your answer using matlab)
   Do the following sum by hand in IEEE double precision computer arithmetic, using the Rounding to Nearest Rule.
   
   \[ (1 + (2^{-51} + 2^{-52} + 2^{-54})) - 1 \]

5. Text exercises on page 17: 14.(c)
   Do the following operation by hand in IEEE double precision computer arithmetic, using the Rounding to Nearest Rule. (check your answer using matlab)
   
   \[ (4.9 - 3.9) - 1 \]

6. Let \( x = 2 \). To avoid subtraction of nearly equal numbers, find an alternative form to evaluate
   
   \[ f(h) = \frac{x^4 - (x-h)^4}{h} \]  
   
   for small \( h \). Compute \( f(h) \) using matlab based on \( f(h) \) in (1) and the alternative form of \( f(h) \) you propose, and report your results for \( h = 10^{-1}, 10^{-2}, \ldots, 10^{-15} \). Summarize and comment on your observations.

7. To find the roots of \( ax^2 + bx + c = 0 \), one can use the quadratic formula
   
   \[ x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \]

   Consider an example with \( a = 1, b = 1.234 \times 10^5 \), and \( c = 4.567 \times 10^3 \).
   
   (a) Use 4-digit, base 10, floating-point arithmetic to compute two approximated roots \( \hat{x}_1 \) and \( \hat{x}_2 \) based on (2). (This means, all intermediate steps also involve 4-digit, base 10, floating-point arithmetic.) In addition, compute the two roots \( x_1 \) and \( x_2 \) using Matlab and regard them as the exact roots. Based on your results, compute the relative errors in \( \hat{x}_1 \) and \( \hat{x}_2 \). Are the 4-digits in \( \hat{x}_1 \) and \( \hat{x}_2 \) accurate?
(b) For the one which has larger relative error, rewrite the root formula based on (0.13) or (0.14) on page 20 of the textbook, and re-compute this approximate root. Still, use 4-digit, base 10, floating-point arithmetic. Now recompute the relative error of this re-computed root.

(Reference reading: Section 0.4)

Instructions:

• Justify your results and answers with sufficient details.

• For computer problems, in addition to the results and/or tables and/or plots (if required in the problem description), you are required to include the codes you write (such as scripts, functions etc). A log of your Matlab session can be included if you find it important to support your results.