AN ANALYSIS OF TWO-PHASE FLOW AND HEAT TRANSFER USING A MULTIDIMENSIONAL, MULTI-FIELD, TWO-FLUID COMPUTATIONAL FLUID DYNAMICS (CFD) MODEL

Richard T. Lahey, Jr.
Donald A. Drew
Center for Multiphase Research
Rensselaer Polytechnic Institute
Troy, NY USA
laheyr@rpi.edu

KEY WORDS
Two-fluid, CFD, two-phase flow, multidimensional

ABSTRACT

This paper reviews the state-of-the-art in the prediction of multidimensional multiphase flow and heat transfer phenomena using a two-fluid model. It is shown that accurate mechanistic CFD predictions are possible for a wide variety of adiabatic and diabatic vapor/liquid and particle/liquid two-phase flows using this computational model.

1. INTRODUCTION

The purpose of this paper is to present a mechanistically-based, four field, two-fluid model for two-phase flow and heat transfer that can accurately predict the distribution of the continuous vapor (cv), continuous liquid (cľ), dispersed vapor (dv), and dispersed liquid (dľ) fields, and is inherently capable of describing the essential features of vapor/liquid flows for different flow regimes. Computational fluid dynamic (CFD) predictions using this model agree with a wide range of experimental data [Lahey, 1995], [Lahey, 1996]. Figure 1 shows a qualitative description of the four fields for a once-through evaporator (i.e., flow boiling in a heated pipe having a subcooled inlet).

The multidimensional, four field, two fluid model is given by:

\[
\frac{\partial}{\partial t} \left( \alpha_{jk} \rho_{k} \right) + \nabla \cdot \left( \alpha_{jk} \rho_{k} \vec{v}_{jk} \right) = \Gamma_{jk} + \dot{m}^{m}_{jk}
\]
where, $\alpha_{jk}$ is the volume fraction of field-j of phase-k, $\Gamma_{jk}$ is the volumetric mass transfer rate due to phase change in field-j of phase-k, and $\dot{m}_{jk}^m$ is the mass source of field-j from other fields of phase-k.

**Momentum Conservation (field-j, phase-k)**

\[
\frac{\partial}{\partial t}(\alpha_{jk} \rho_k \vec{v}_{jk}) + \nabla \cdot \left( \alpha_{jk} \rho_k \vec{v}_{jk} \times \vec{v}_{jk} \right) + \nabla \cdot (\alpha_{jk} \rho_k \vec{v}_{jk} \times \vec{v}_{jk}) + \nabla (\alpha_{jk} \rho_k \vec{v}_{jk}) = -\nabla \cdot \left( \alpha_{jk} \rho_k \sigma_{jk} \right) + \dot{M}_{jk} - \dot{M}_{jk}\mu = \Gamma_{jk} \nu_j + \dot{m}_{jk}^m \nu_{jk} \tag{2}
\]

where $\vec{v}_{jk}$ is the Reynolds stress for field-j of phase-k and $\dot{M}_{jk}$, $\dot{M}_{jk}\mu$ are the interfacial and wall forces (per unit volume), respectively, and as discussed in Section 2.1, $\nu_j = \nu_{jk}^i - \nu_{jk}^j$. It should be noted that, in general, $\tau_{jk}^T = -\rho_k \nu_{jk} \nu_{jk} + \tau_{jk}^C$ where, $\tau_{jk}^C$ is the induced shear due to particle collisions in solid/liquid flows [Alajbegovic et al, 1999].

**Energy Conservation (field-j, phase-k)**

\[
\frac{\partial}{\partial t}(\alpha_{jk} \rho_k \vec{h}_{jk}) + \nabla \cdot \left( \alpha_{jk} \rho_k \vec{h}_{jk} \times \vec{h}_{jk} \right) + \nabla \cdot (\alpha_{jk} \rho_k \vec{h}_{jk} \times \vec{h}_{jk}) + \nabla (\alpha_{jk} \rho_k \vec{h}_{jk}) = -\nabla \cdot \left( \alpha_{jk} \rho_k \vec{h}_{jk} \times \vec{h}_{jk} \right) - \frac{\dot{D}_{jk}}{\partial t} - q_{jk}^s - D_{jk} - \rho_{jk} A_{jk} = \Gamma_{jk} \mu_j + \dot{m}_{jk}^m \mu_{jk} \tag{3}
\]

where,

\[
\mu = h - p / \rho \quad \text{(internal energy)} \tag{4a}
\]

\[
q_{jk}^s = -h_{jk} \nu_{jk} \quad \text{(turbulent heat flux)} \tag{4b}
\]

\[
q_{jk}^s = \alpha_{jk} \rho_k \vec{v}_{jk} \times \vec{v}_{jk} \quad \text{(interfacial heat flux)} \tag{4c}
\]

\[
\dot{D}_{jk} \triangleq \frac{\nabla \cdot \vec{v}_{jk}}{\partial \nu_{jk}} \quad \text{(dissipation)} \tag{4d}
\]

\[
p_{jk} \triangleq \frac{\nabla \cdot \vec{v}_{jk}}{\partial \nu_{jk}} \quad \text{(static pressure)} \tag{4e}
\]

It has been found [Lopez de Bertodano et al, 1994a] that k-ε models, which have been widely used in CFD codes to describe turbulence in single-phase flows, can be extended to two-phase flows. For example, in bubbly flows the turbulence transport equations for the continuous liquid (c/l) field are given by:
Turbulent Kinetic Energy

\[
\alpha_{cf} \frac{Dk_{cf}}{Dt} = \nabla \cdot \left[ \alpha_{cf} \left( \frac{u_{cf}^T}{\sigma_k} + \nu \right) \nabla k_{cf} \right] + \alpha_{cf} \left[ P_{cf} - \epsilon_{cf} \right] + \alpha_{cf} \Phi_k \tag{5a}
\]

where,

\[
k_{cf} = \frac{1}{2} \frac{\nabla \cdot \nu_{cf}}{\nu_{cf}}
\]

Turbulent Dissipation

\[
\alpha_{cf} \frac{D\epsilon_{cf}}{Dt} = \nabla \cdot \left[ \alpha_{cf} \left( \frac{u_{cf}^T}{\sigma_\epsilon} + \nu \right) \nabla \epsilon_{cf} \right] + \alpha_{cf} C_{\epsilon_1} \frac{P_{cf}\epsilon_{cf}}{k_{cf}} - \alpha_{cf} C_{\epsilon_2}^2 \frac{\epsilon_{cf}^2}{k_{cf}} + \alpha_{cf} \Phi_\epsilon \tag{5b}
\]

where [Sato et al, 1975] gives the two-phase turbulent viscosity as:

\[
u_{cf}^T = C_\mu \frac{k_{cf}^2}{\epsilon_{cf}} + 1.2 R_{dv} \alpha_{dv} |\nabla r| \tag{6}
\]

and,

\[\nabla r = \nabla_{dv} - \nabla_{cf} .\]

For bubbly flows, the source term for turbulent kinetic energy, \( \Phi_k \) is [Lee et al, 1989], [Varaksin & Zaichik, 2000]:

\[
\Phi_k = \frac{k_{cf}}{C_{\epsilon_2} \epsilon_{cf}} \Phi_\epsilon = C_p (1 + C_D^4) \alpha_{dv} \frac{|\nabla r|^3}{D_{dv}}, \tag{7}
\]

where \( C_p = 0.25 \) for potential flow around a sphere [Lopez de Bertodano, 1992].

Similar turbulent transport equations can be written for the dispersed phase [Alajbegovic et al., 1999].
2. DISCUSSION

2.1 Closure

In order to achieve closure we must be able to express all the parameters and variables in Eqs. (1)-(7) in terms of the state variables (i.e., the dependent variables) of the two-fluid model. For example, the interfacial jump condition associated with phase change is given by:

\[ \Gamma_{jk} = \sum_{jk} \Gamma_{jk - jk} \]  

(8a)

where \( \Gamma_{jk - jk} \) is the volumetric mass transfer rate due to phase change from field- \( j \) of phase- \( k \) to field-\( j \) of phase \( k \), and the summation is over all fields of phase- \( \tilde{k} \), where \( \tilde{k} \) is not \( k \) (i.e., if \( k \) denotes vapor, then \( \tilde{k} \) denotes liquid, and vice versa). The jump conditions for mass transfer due to phase change are then:

\[ \Gamma_{jk - jk} + \Gamma_{j\tilde{k} - jk} = 0 \]  

(8b)

across the interface between field- \( j \) of phase- \( \tilde{k} \) and field-\( j \) of phase \( k \).

Similarly, for momentum jump we note that,

\[ M_{jk} = \sum_{jk} M_{jk - jk} \]  

(8c)

where \( M_{jk - jk} \) is the force per unit volume on field-\( j \) of phase- \( k \) from field- \( \tilde{j} \) of phase \( \tilde{k} \). For this phase change example, the momentum jump conditions are:

\[ M_{jk - jk} + M_{j\tilde{k} - jk} + \Gamma_{jk - jk} \mathbf{v}^{j}_{jk - jk} + \Gamma_{j\tilde{k} - jk} \mathbf{v}^{j}_{j\tilde{k} - jk} = M_{j\tilde{k} - jk}^{\prime \prime} \]  

(8d)

where \( M_{j\tilde{k} - jk}^{\prime \prime} \) is the interfacial source density of momentum, which for bubbly flow is:

\[ M_{j\tilde{k} - jk}^{\prime \prime} = \nabla \cdot \left[ \alpha_{dv} \mathbf{\tau} \right] \]

where \( \mathbf{\tau} \) is defined in Eq. (26).

For energy jump, we note that,

\[ q^{\prime \prime}_{jk} \cdot \mathbf{A}_{jk} = \sum_{jk} q^{\prime \prime}_{jk - jk} \cdot \mathbf{A}_{jk - jk} \]  

(8f)
where \( q^{\prime\prime}_{jk-jk}, \cdot A^{m}_{jk-jk} \) is the interfacial heat transfer (per unit volume) to field-\( j \) of phase-\( k \) from field-\( j' \) of phase-\( k' \).

The energy jump conditions associated with phase change are thus:

\[
q^{\prime\prime}_{jk-jk}, \cdot A^{m}_{jk-jk} + q^{\prime\prime}_{jk-jk}, \cdot A^{m}_{jk-jk} + \Gamma_{jk-jk} e^{l}_{jk-jk} + \Gamma_{jk-jk} e^{l}_{jk-jk} + p_{jk-jk} \frac{\partial \alpha_{jk}}{\partial t} + p_{jk-jk} \frac{\partial \alpha_{jk}}{\partial t} = E_{jk-jk}^{i}
\]

where \( E_{jk-jk}^{i} \) is an interfacial source term density for energy.

It is conventional to partition the liquid interfacial force density into drag (D) and non-drag (ND) terms. For example, for bubbly flow we have:

\[
M_{cf} = M_{cf}^{(D)} + M_{cf}^{(ND)}
\]

For the drag force we can write:

\[
M_{cf}^{(D)} = -M_{cf}^{(D)} = -\frac{1}{8} \rho_{cf} C_{D} \sum \Lambda_{i}
\]

where \( C_{D} \) is an appropriate drag coefficient.

To derive an expression for the interfacial area density, \( \Lambda_{i} \), let us consider the Boltzmann transport equation for the dispersed phase (dv). In particular, the Boltzmann transport equation for bubbles of volume \( u \) is given by [Valenti et al., 1991]:

\[
\frac{\partial f}{\partial t} + \nabla \cdot vf = \frac{df_c}{dt} + \frac{df_b}{dt}
\]

where \( f_{du} \) is the probability of having a bubble of volume \( u \) between \( u \) and \( u + du \), and the coalescence (c) and breakup (b) terms are given by [Drew & Passman, 1998]:

\[
\frac{df}{dt} \bigg|_{c} = -\int_{0}^{\infty} c(u',u) f(u',\bar{x},t) f(u,\bar{x},t) du' + \frac{1}{2} \int_{0}^{\infty} c(u',u-u') f(u',\bar{x},t) f(u-u',\bar{x},t) du'
\]

(12a)
and,
\[
\frac{df}{dt} \bigg|_b = -\int_{0}^\infty b(u',u)f(u,x,t)du' + \int_{0}^u b(u',u-u')f(u',x,t)du'
\]
(12b)

Here, \(c\) and \(b\) are the coalescence and breakup rate kernels, respectively.

If we multiply Eq. (11) by the area of a bubble of volume \(u\), \(A_u = 4\pi \left(\frac{3u}{4\pi}\right)^{2/3}\), and integrate over all volumes, \(u\), we obtain [Drew & Passman, 1998]:
\[
\frac{\partial A}{\partial t} + \nabla \cdot (A_i v_i) = S_A
\]
(13)

where \(S_A\) is the generalized interfacial area source/sink term, which for bubbly flow is:
\[
S_A = 4\pi \int_{0}^\infty \left[ \frac{df}{dt} \bigg|_c + \frac{df}{dt} \bigg|_b \right] \left(\frac{3u}{4\pi}\right)^{2/3} du
\]
(14)

Formulations for \(S_A\) have been proposed by Millies et al [1996] and Wu et al [1998] for a two field, two-fluid model, and this approach has been applied to bubble clusters for the analysis of the bubbly/slug flow regime transition [Kalkach-Navarro et al, 1994].

The wall force on the dispersed vapor field, \(M_{dv}^w\), is given by [Antal et al, 1991], [Lopez de Bertodano, 1992]:
\[
M_{dv}^w = -M_{c\ell}^w = \text{Max} \left\{ 0, \frac{\alpha_{dv} \rho_{c\ell} v_{axial} \cdot v_{axial}}{R_{dv}} \left[ C_{w1} + C_{w2} \left( \frac{R_{dv}}{y} \right) \right] \delta_w n_w \right\}
\]
\[
- C_{w1} \frac{D_{c\ell}}{10R_{dv}} \alpha_{dv} v_{c\ell}^* \sqrt{\frac{\delta_w}{R_{dv}}} - M_{wc}^d
\]
(15)

where, \(n_w\) is a unit vector which is normal to the wall of the conduit, and,
\[
y = (x - x_w) \cdot n_w
\]
\[
v_{axial} = v_r - [n_w \cdot v_r] n_w
\]
\[
C_{w1} = -0.104 - 0.06 \left| v_r \right|
\]
\[
C_{w2} = 0.147
\]
\[
\delta_w = \begin{cases} 1.0, y \leq 2R_{dv} \\ 0.0, \text{otherwise} \end{cases}
\]
and, $M_{d}^{wc}$ is the wall force induced on the particles which strike the wall in solid/liquid two-phase flows [Alajbegovic et al, 1999].

The first term on the right hand side (RHS) of Eq. (15) is a force which is normal to the wall and the second term on the RHS is a force parallel to the wall. These forces are important near the wall.

It should be noted that in accordance with standard practice for single-phase flow CFD evaluations, the Law of the Wall was used rather than a no slip boundary condition. This saves significant computer time and its validity for two-phase flows has been verified by Marie’ [1987] and Vassallo [1999] and Chahed & Masbernat [1999]. The specific implementation of the two-phase Law of the Wall has been given by [Lopez de Bertodano et al, 1994b] and thus will not be repeated here.

Next, we may use inviscid flow theory to derive many of the interfacial transfer laws. In particular, we may solve for the potential flow velocity field around the dispersed phase using,

$$\nabla^2 \phi = 0$$  \hspace{1cm} (16a)

We then obtain the velocity from,

$$\mathbf{v}_{cf} = -\nabla \phi$$  \hspace{1cm} (16b)

and the pressure field by using the Bernoulli Equation,

$$p - p_o = -\rho_{cf} \left[ \frac{\partial \phi}{\partial t} + |\nabla \phi|^2 \right]$$  \hspace{1cm} (16c)

For the particular case of bubbly flow, this analysis yields [Ruggles et al, 1989], [Drew & Passman, 1998]:

$$p_{cl_i} = p_{cl} - (1 - \alpha_{dv}) C_p \rho_{cl} \mathbf{v}_{r} \cdot \mathbf{v}_{r} - \frac{1}{2} \rho_{cl} (k_{cl} + k_{dv}) + \frac{1}{2} \rho_{cl} \left[ R^2 \frac{\dot{R}}{R} + 3/2 \dot{R}^2 \right]$$  \hspace{1cm} (17)

where, $\dot{R} = \frac{Dv}{Dt}$, and for spherical bubbles,

$$p_{dv_i} = p_{cl_i} = \frac{2\sigma}{R_b} + 4\mu_{cl} \frac{\dot{R}}{R_b}$$  \hspace{1cm} (18)
It should be noted that if the bubbles do not grow or shrink, Eq. (17) reduces to the well known quasi-static closure law:

$$p_{ce} - p_{ct} = - (1 - \alpha_{dt}) C_{P} \rho_{ct} \nabla \cdot \mathbf{v} - \frac{1}{2} \rho_{ct} (k_{ct} + k_{dc})$$

(19)

We may also ensemble average the pressure at the interface to obtain the interfacial force density, $M_{ce}^{(nd)}$. Using Eqs. (16) Arnold et al [1989], Lopez de Bertodano [1992] and Drew & Passman [1998] have shown that for spherical bubbles:

$$M_{ce}^{(nd)} = - \rho_{ct} \nabla \alpha_{dv} + \alpha_{dv} \rho_{ct} C_{vm} \left[ \frac{D \nabla_{dv}}{Dt} - \frac{D \nabla_{ct}}{Dt} \right]$$

$$+ \alpha_{dv} \rho_{ct} C_{L} \nabla \cdot \mathbf{v} + \alpha_{dv} \rho_{ct} C_{2} \left[ \nabla \cdot \left( \nabla \frac{\mathbf{v}}{\mathbf{v}} + \nabla \nabla \right) + \nabla \cdot \nabla \right]$$

$$+ b_{s} \rho_{ct} \nabla \alpha_{dv} + a_{s} \rho_{ct} \nabla \alpha_{dv} + \alpha_{dv} \rho_{ct} C_{rot} \nabla \nabla$$

$$+ \alpha_{dv} \rho_{ct} C_{L} \nabla \alpha_{dv} + \rho_{ct} \nabla \alpha_{dv} \cdot \left[ a_{d} \nabla \frac{\mathbf{v}}{\mathbf{v}} + b_{d} \left| \nabla \frac{\mathbf{v}}{\mathbf{v}} \right| \right]$$

$$+ a_{f} \rho_{ct} \left( k_{ct} - C_{p} \alpha_{dv} \left| \nabla \frac{\mathbf{v}}{\mathbf{v}} \right| \right)$$

$$+ b_{f} \frac{\rho_{ct} \tau}{\rho_{dv}} \nabla \alpha_{dv}$$

(20)

We note that the second term on the right hand side of this equation is the virtual mass force, the fifth from the last term is often called the lateral lift force [Drew et al, 1990], the next to the last term is the turbulent dispersion term, and the last terms arise due to the concentration gradients [Drew & Passman, 1998]. It should be noted that the bubble’s turbulent dispersion force was phenomenologically derived by Lopez de Bertodano [1992], where $C_{TD}$ is a free parameter which various investigators have found to be in the range 0.1 to 1.0. Other forms of the turbulent dispersion force have been derived [Drew & Passman, 1998] but they will not be discussed herein. It should be noted that $C_{TD} = 1.0$ was used in the calculations presented in the next section, however, more accurate expressions for $C_{TD}$ is given by Eq. (22a) and for $C_{L}$ by Eq. (22b).

It is significant to note that there are no arbitrary constants in this model, that is, for spherical bubbles:

$$C_{vm} = C_{L} + C_{rot} = \frac{1}{2}, \quad C_{L} = C_{rot} = \frac{1}{4}, \quad C_{1} = \frac{5}{4}, \quad C_{p} = \frac{1}{4}, \quad C_{2} = \frac{9}{20}, \quad b = b = b = \frac{3}{20}, \quad a_{d} = \frac{1}{20}, \quad a_{f} = \frac{2}{5}$$

(21)
The turbulent dispersion coefficient, $C_{TD}$, has been determined [Moraga et al, 2000] from DNS and experimental data to be,

$$C_{TD} = \frac{3C_D |v_r|^T}{8R_{dv}k_{cl}Sc_{dv}}$$

(22a)

where the Schmidt number is, $Sc_{dv} = 0.833$.

It should also be noted that in real fluids, $C_p \geq 1/4$ [Lopez de Bertodano et al, 1994b], [Park et al, 1998], and the lateral lift coefficient, $C_L$, is a function of Reynolds number. In particular, it may be given by [Moraga et al, 1999]:

$$C_L = [0.12 - 0.2 \exp(-ReRe_v/36 \times 10^4)] \exp[ReRe_v/3 \times 10^7]$$

(22b)

where,

$$Re = \frac{v_r D_{dv}}{\omega_{cl}}$$

$$Re_v = \frac{\omega D_{dv}^2}{\omega_{cl}}$$

and $\omega$ is the local vorticity (i.e., velocity gradient) of the continuous (liquid) phase. It is interesting to note that Eq (22) has a sign reversal at large enough Reynolds numbers, and for most of the calculations presented herein this correlation yields $C_L \approx 0.1$.

For dilute two-phase flows the Reynolds stress tensor for the continuous phase can be determined by superposition of the shear-induced (SI) and particle-induced (PI) Reynolds stresses [Theofanous & Sullivan, 1982], [Lopez de Bertodano et al., 1994b]:

$$\tau_{cl}^T = \tau_{cl}^T(SI) + \tau_{cl}^T(PI)$$

(23a)

and [Drew & Passman, 1998]:

$$\tau_{dv}^T = \frac{\rho_{dv}}{\rho_{cl}} \left(1 - e^{-\theta_{dv}/\theta_{dv}}\right)^2$$

(23b)

where, the relaxation time of the bubbles and liquid eddies are, respectively,

$$\theta_{dv} = \frac{\rho_{dv} V_{dv}}{6\pi \mu_{cl} R_{dv}} = \frac{2}{9\mu_{cl}} \frac{\rho_{dv} R_{dv}^2}{\rho_{cl}}$$

(23c)

and,

$$\theta_{cl} = k_{cl} / \varepsilon_{cl}.$$ 

Superposition has been verified experimentally for dilute bubbly flows by Lance & Bataille [1991], for $\alpha \leq 1\%$, and Theofanous & Sullivan [1982], for $\alpha \leq 10\%$. 

9
Using inviscid flow theory and cell model averaging [Nigmatulin, 1979], we obtain:

\[ \alpha_{ct} \tau_{ct} = -\alpha_{dv} \rho_{ct} \left[ a_{ct} \mathbf{v}_r + b_{ct} \mathbf{v}_r \cdot \mathbf{v}_r \right] \]  
(24)

where, for spherical bubbles, \( a_{ct} = \frac{1}{20} \), and \( b_{ct} = \frac{3}{20} \).

The shear-induced Reynolds stress of the continuous phase [Drew & Passman, 1998],

\[ \frac{\tau_{ct}}{\rho_{ct}} = -\mathbf{v}_{ct} \cdot \mathbf{v}_{ct} = \left\{ -\frac{2}{3} k_{ct} A + \mathbf{v}_{ct} \mathbf{v}_{ct} \right\} \]

\[ + \frac{4b_0}{c_{ct}} \alpha_{dv} \left[ k_{ct} - C_0 \alpha_{dv} \left| \mathbf{v}_r \right|^2 \right] \left( \mathbf{I} \right) \left[ \left( 1 - e^{-\omega_{ct}/\tau_{ct}} \right)^2 \right] \]

can be evaluated using the \( k_{ct} \) and the \( \mathbf{v}_{ct} \) from Eqs. (5a), (6) and (23b), and an algebraic stress law given by [Rodi, 1984]:

\[ \mathbf{A}_{ct} = \begin{bmatrix}
\frac{(C_i + \kappa + \beta - 1)}{C_i} & 3 \frac{3}{2C_i} \left( \mathbf{v}_i \mathbf{\nabla}_{\mathbf{v}_1} \frac{\partial \mathbf{v}_3}{\partial \mathbf{x}_1} \right) \mathbf{e}_{ct} & 3 \frac{3}{2C_i} \left( \mathbf{v}_1 \mathbf{\nabla}_{\mathbf{v}_1} \frac{\partial \mathbf{v}_3}{\partial \mathbf{x}_1} \right) \mathbf{e}_{ct} & 0 \\
0 & \frac{(C_i + \kappa + \beta - 1)}{C_i} & 3 \frac{3}{2C_i} \left( \mathbf{v}_1 \mathbf{\nabla}_{\mathbf{v}_1} \frac{\partial \mathbf{v}_3}{\partial \mathbf{x}_1} \right) \mathbf{e}_{ct} & 0 \\
0 & 0 & -\frac{2(\kappa + \beta - 1)}{C_i} + 1 - \frac{3}{C_i} \left( \mathbf{v}_1 \mathbf{\nabla}_{\mathbf{v}_1} \frac{\partial \mathbf{v}_3}{\partial \mathbf{x}_1} \right) \mathbf{e}_{ct}
\end{bmatrix} \]

(25)

where, \( \beta = 0.1091, \kappa = 0.7636, \) and \( C_{f} = 1.5 \) [Lahey et al, 1993], and the various Reynolds Stresses in the liquid are evaluated assuming [Lopez de Bertodano, 1992],

\[ -\mathbf{v}_i \mathbf{v}_j = -\frac{2}{3} k_{ct} \mathbf{I} + \mathbf{v}_T \left[ \mathbf{\nabla}_{\mathbf{v}_{ct}} + \mathbf{\nabla}_{\mathbf{v}_T} \mathbf{v} \right]. \]

Finally, it should be noted that the force required to hold the bubbles spherical results in a stress in the dispersed phase given by [Park et al, 1998], [Drew & Passman, 1998]:
For solid/liquid two-phase flows, we must also model the effect of particle/particle and particle/wall collisions. Figure-2 is a schematic of a binary collision. The resultant collision-induced shear stress is given by [Alajbegovic et al, 1999]:

\[
\tau_{\text{col}}^{\cd} = \rho_{\text{ef}} \left[ a \mathbf{v}_d \cdot \mathbf{v}_d + b \left| \mathbf{v}_d \right|^2 \right] + (p_{\text{atm}} - p_{\text{ef}}) \mathbf{I}
\]

\[
+ b_f \left( \frac{P_{\text{ef}}}{\rho_{\text{df}}} \tau_{\text{col}}^{\cd} + a_f \rho_{\text{ef}} \left[ k_{\text{ef}} - C_{\rho} \alpha_{\text{ef}} \left| \mathbf{v}_d \right|^2 \right] \mathbf{I} \right)
\]

(26)

Note that the first term on the LHS of Eq. (27) is the well-known isotropic result of Chapman and Cowling [1970], and the other terms represent nonisotropy effects.

Similarly, Figure-3 is a schematic showing particle/wall interaction. The resultant wall force of the dispersed phase due to inelastic collisions is, for \( r < 2R_d \), given by Alajbegovic et al [1999] as:

\[
M_{\text{wc}}^d = 6\rho_d \left( 1 + e_w \right) \left( r_m \right)^{\frac{n}{2}} D_d (1 + e_w) \left( \mathbf{v}_d \cdot \mathbf{v}_d \right) + \left( r - R_d \right) \left( \mathbf{v}_d \cdot \mathbf{v}_d \right) \frac{\partial \alpha_d}{\partial t}
\]

\[
+ r \alpha_d \left( \mathbf{v}_d \cdot \mathbf{v}_d \right) + 2 \sqrt{1 + e_w^2} \frac{\partial \mathbf{v}_d}{\partial t}
\]

\[
\times r \alpha_d \sqrt{\left( \mathbf{v}_d \cdot \mathbf{v}_d \right)} \frac{\partial \mathbf{v}_d}{\partial t}
\]

(28)
where, \( r'_m = \sqrt{r(2R_d - r)} \)

\[ e_w = \text{the wall's coefficient of restitution.} \]

Significantly, the two-fluid model presented herein is fully self-consistent, is well-posed [Park et al, 1998], and unlike many other two-fluid models, satisfies the Second Law of Thermodynamics [Arnold et al, 1990]. Next, let us consider the assessment of this model against some of the available experimental data.

### 2.2 Model Assessment (Steady State)

The two-fluid model has been evaluated using various CFD solvers (eg, PHOENICS and CFX), and good agreement with the available experimental data was found without “tuning” any of the parameters in the model.

Figures 4 to 6 show good agreement with the bubbly air/water upflow pipe data of Serizawa [1974]. Note that the model properly predicts the peaking of the dispersed vapor volume fraction \( (\alpha_{dv}) \) near the wall of the conduit. Similarly, Figure 6 shows that the same model predicts the bubbly air/water downflow pipe data of Wang et al [1987], in which the dispersed vapor volume fraction peaks at the center of the vertical pipe. This is a direct consequence of a change in the sign of the lateral lift force in Eq. (22).

Figures 8 shows that the two-fluid model is also capable of predicting data in a complex geometry conduit. In particular, the model is able to predict the air/water bubbly upflow data of Lopez de Bertodano [1992] which was taken in an isosceles triangle.

Not all two-phase flows of interest are conduit flows. For example, a plunging liquid jet can entrain air bubbles resulting in a spreading two-phase jet in a liquid pool [Bonetto et al, 1993]. Significantly, as can be seen in figures 9, the same multidimensional four field, two-fluid model which predicts various conduit flows, also predicts spreading two-phase jet data (i.e., a "free field" two-phase flow). Moreover, as can be seen in figures 10, this model also predicts external two-phase flows over solid surfaces, and it has been used to successfully predict two-phase flow phenomena around naval surface ships [Carrica et al, 1999]; in particular, the effect of entrained air bubbles on the formation of ship wakes.

It should be clear that the multidimensional, four field, two-fluid model is capable of predicting a wide variety of adiabatic two-phase flows. Similarly, once the proper constitutive laws for interfacial heat transfer have been specified, the same two-fluid model can also predict diabatic two-phase flows. In particular, as can be seen in Figure 11, this model is able to predict [Lahey, 1996], the R-113 subcooled boiling data of Velidandla et al [1995], which were taken in an internally heated annular test section. Moreover, as shown in Figure 12, it can also predict boiling SUVA data taken in a heated
rectangular test section [Lahey, 1998]. We note in Figure 12 that numerous flow regimes, and their transition, are predicted, and the predictions of the two-phase pressure drop does not require any separate empirical correlations (i.e., the predicted wall shear was used to determine the pressure drop).

In addition, the same two-fluid model has been found to be able to predict phase separation in branching conduits [Lahey, 1992]. For example, Figures 13 to 16 demonstrate the model’s predictive capabilities for a Tee junction in which the side branch has three different orientations. It should be noted in figure 16 that the two-phase pressure drop was predicted without the need for empirical correlations (i.e., the predicted wall shear yields the two-phase friction pressure drop).

CFD evaluations using a multidimensional two-fluid model represents a significant breakthrough in our ability to analyze multiphase flows and clearly demonstrates the usefulness of mechanistic, multidimensional two-fluid models for phase change system design and evaluations.

While the discussions given above have focused on bubbly flow (with a spherical dispersed phase), similar predictive capabilities are possible for other flow regimes (e.g., see Figure 12) using a multidimensional, four field, two-fluid model and the appropriate, flow regime specific, closure laws [Antal et al, 1998].

Let us next consider the predictive capabilities of the multidimensional two-fluid model discussed herein for solid/liquid systems. For systems of this type, particle/particle and particle/wall interactions must be modelled, as in Eqs. (27) and (28).

Alajbegovic et al [1999] used this model to predict the solid/liquid data of Alajbegovic et al [1994] and Assad [1995]. These data consisted of positive, negative and neutral buoyant spheres having the properties given in Table-I.

<table>
<thead>
<tr>
<th>Material</th>
<th>Specific Gravity</th>
<th>Diameter (mm)</th>
<th>Buoyancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expanded Polystyrene</td>
<td>0.032</td>
<td>1.79</td>
<td>Positive</td>
</tr>
<tr>
<td>Ceramic</td>
<td>2.45</td>
<td>2.32</td>
<td>Negative</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>1.030</td>
<td>1.96</td>
<td>Neutral</td>
</tr>
</tbody>
</table>

Figure-17 show that the two-fluid model predicts the observed wall peaking in the particle volume fraction, as well as the mean velocity and turbulence fields, for positive buoyant particles. Figures-18 show similar results for negative buoyant particles. It can
be seen that the observed interior coring of the particles is predicted, as is the velocity field. Finally, Figures-19 show the corresponding predictions for neutral buoyant particles. It is interesting to note that neutral buoyant particles simulate bubbly vapor/liquid flows at microgravity. Indeed, the predicted profiles are quite similar to those measured by Kamp [1996] for bubbly flows in microgravity.

2.3 Model Assessment (Transients)

The previous assessments were for steady flow. Let us next consider the ability of the two-fluid model to predict transient phenomena. The multidimensional, four field, two-fluid model presented previously can be integrated in the lateral direction to give the corresponding one dimensional two-fluid model. This can be written in matrix form as:

\[
\begin{bmatrix}
\frac{\partial u}{\partial t} + \frac{\partial u}{\partial z} = C \frac{u}{C} + 2
\end{bmatrix}
\]

If the two-fluid model is valid for transients one should also be able to predict wave propagation phenomena in two-phase flows.

Equation (27) may be linearized, resulting in:

\[
A \frac{\partial \delta u}{\partial t} + B \frac{\partial \delta u}{\partial z} = C \frac{\delta u}{C}
\]

where,

\[
C' \frac{\delta u}{C} = \left[ C \frac{\delta u}{C} + u \left( \frac{\partial C}{\partial u} \right) - \left( \frac{\partial B}{\partial u} \right) \left( \frac{\partial u}{\partial z} \right) \right]
\]

Next we assume a perturbation of the form,

\[
\delta u = u' e^{i(kz-\omega t)}
\]

where \( k = \frac{2\pi}{\lambda} \) is the wave number and \( \omega = 2\pi f \) is the angular frequency of the system.

Combining Eqs. (28) and (29) we obtain,

\[
\left[ -i\omega A - ik B C' \right] u' = 0
\]

Since \( u' \neq 0 \), the so-called dispersion relation of the two-fluid model is given by:
It is interesting to note that two of the roots of Eq. (31) yield the two-phase sonic velocity ($C_{2\phi}$):

$$C_{2\phi} = \frac{\left[\frac{\omega}{\text{Re} (k)}\right]^+ - \left[\frac{\omega}{\text{Re} (k)}\right]^-}{2}$$

As can be seen in Figures 20 to 25, the two-fluid model agrees quite well with the available data for sonic wave propagation in bubbly two-phase flows.

It can also be noted that the high frequency limit (i.e., $k \to \infty$) of Eq. (31) yields,

$$\det \left[ A_\omega \left( \frac{\omega}{k} \right) - \frac{V}{k} B_{l_0} \right] = 0$$

where the roots, ($\omega/k$), can be recognized as being the eigenvalues of the system of equations. When a critical flow condition exists one of the roots of Eq. (33) will vanish (i.e., $\left[\frac{\omega}{k}\right]^p = u_{2\phi} - C_{2\phi} = 0$), thus the choking condition is given by,

$$\det \left[ B_{l_0} \right] = 0$$

It has been found [Ruggles et al, 1989] that Eq. (34) predicts critical flow, without the need for separate critical flow models.

Similarly, two other roots of Eq. (31) yield the void waves celerities, $C_{\alpha}^+$. It has been found that these roots agree with the available void wave data [Park et al, 1998] which further supports the physical basis of the two-fluid model. Finally, a nonlinear void wave analysis has been performed using Eq. (29a) and it has been shown that the experimentally observed solitons and breaking void waves are also well predicted by the two-fluid model [Lahey, 1991], [Park et al, 1998].

3. CONCLUSION

It appears that a properly formulated two-fluid model can predict a wide variety of steady and transient multiphase flow phenomena. Moreover, data of the type discussed herein can be used to assess the closure laws used in two-fluid models for various flow regimes. It is hoped that this paper will motivate other researchers to use,
and further develop, multidimensional CFD methodology for the analysis of multiphase flow and heat transfer phenomena in systems and processes of practical concern.

**NOMENCLATURE**

- \( A_i'''' \) = Interfacial area density (1/m)
- \( R_b \) = Bubble radius (D/dv/2)
- \( \alpha_{jk} \) = Volume fraction of field-j of phase-k
- \( q_{jk}'''' \) = Volumetric heat source in field-j of phase-k
- \( h_{jk} \) = Enthalpy of field-j of phase-k
- \( \tau_{jk} \) = Viscous shear stress tensor of field-j of phase-k
- \( k_{jk} \) = Turbulent kinetic energy of field-j of phase-k
- \( \mu_{jk} \) = Internal energy of field-j of phase-k
- \( M_{jk} \) = Interfacial force density of field-j of phase-k
- \( \varepsilon_{jk} \) = Turbulent dissipation of field-j of phase-k
- \( \rho_k \) = Density of phase-k
- \( P_{jk} \) = Turbulence production in field-j of phase-k
- \( \nabla_{jk} \) = Velocity of field-j of phase-k
- \( \nabla'_{jk} \) = Velocity fluctuations in field-j of phase-k
- \( p_{jk} \) = Static pressure of field-j of phase-k
- \( \Delta p \) = Pressure drop
- \( \tau_{jk}^T \) = Reynolds stress of field-j of phase-k
- \( v \) = Bubble volume
- \( u \) = Bubble volume
- \( g \) = Gravity
REFERENCES


Figure 1. Typical Model Four Field Model Predictions
Figure 2. Particle-Particle Interaction.
Figure 3. Particle/Wall Interaction
Figure 4. Comparison with Serizawa’s Data [1974]: Void Fraction Distribution

Figure 5. Comparison with Serizawa’s Data [1974]: Velocity Fluctuation Distributions

Figure 6. Comparison with Serizawa’s Data [1974]: Reynolds Stress Distribution.

Figure 7. Comparison with Wang’s Downflow Data [1987]: Void Fraction Distribution
Figure 8a. Comparison with Lopez de Bertodano’s Data [1992]: Void Fraction Distribution

Figure 8b. Comparison with Lopez de Bertodano’s Data [1992]: Average Axial Velocity Distribution
Figure 9a. Void Fraction Profiles [Bonetto et al., 1993]: Measured (Symbols) and Predicted (Lines)

Figure 9b. Liquid Velocity Profiles [Bonetto et al., 1993]: Measured (Circles) and Predicted (Line)
Figure 10a. Bubbly Two-Phase flow Over a Flat Plate [Moursali et al, 1995]

Figure 10b. The Prediction of the Near-Wall Void Distribution (at x=1.0m)
Figure 11. Predictions for Subcooled Boiling R-113 in an Annulus [Velidandla et al., 1995]
Figure 12. Comparison of Four Field Model Predictions with Experimental Results for Heated SUVA Flow in a High Aspect Ratio Duct [Lahey, 1998]
Figure 13. Phase Separation for a Horizontal Branch

Figure 14. Phase Separation for a Vertical Branch Above Junction

Figure 15. Phase Separation for a Vertical Branch Below Junction

Figure 16. Pressure Drop in a Tee Junction
Figure 17. Prediction of data for positive buoyant particles [Alajbegovic et al., 1999]
Figure 18. Prediction of data for negative buoyant particles [Alajbegovic et al, 1999]
Figure 19. Prediction of data for neutral buoyant particles [Alajbegovic et al, 1999]
Figure 20. Sound Speed vs. Frequency (Effect of Bubble Size) [Ruggles, 1987]

Figure 21. Sound Speed vs. Frequency (Effect of Void Fraction) [Ruggles, 1987]
Figure 22. Attenuation vs. Frequency [Ruggles, 1987]

Figure 23. Pressure Pulse Propagation Speed vs. Global Void Fraction [Ruggles, 1987]
Figure 24. Sound Speed vs. Frequency (Data of Silberman [1957])

Figure 25. Attenuation vs. Frequency (Data of Silberman [1957])