Stochastic processes entails a dynamical approach to probability theory, with uncertainty entering a system over time.

Mathematically a stochastic process can be thought of as a random function that maps a parameter domain (time domain) into a state space.

\[ X: T \rightarrow S \]

\[ \text{parameter domain} \rightarrow \text{state space} \]

The study of stochastic processes is typically decomposed into frameworks where each of these spaces is discrete or continuous.

Simplest setting is discrete parameter domain and discrete state space.

Within this framework falls discrete-time Markov chains, with both finite and countable state space.

What does the discrete time parameter measure? Generally it refers to epochs which are just convenient times at which to observe or characterize the system. Typically these are regularly spaced time intervals, though not strictly necessary.

What could a finite state space represent?
Sometimes it is just a finite range of spatial positions. Often it is a description from a finite set of categories:

- Epidemiology: susceptible, latently infected, actively infective, recovered, dead, immune; extend to a finite population. (see EpiSims)
- Credit states of an obligor: good, dicey, dangerous, default
- Conformational states of a biomolecule
• Credit states of an obligor: good, dicey, dangerous, default
• Conformational states of a biomolecule

What could a countably infinite state space represent?

\[ S = \mathbb{Z} \quad \text{or} \quad S = \mathbb{Z}_{\geq 0} \]

Such state spaces typically model counting the population of some object with no definite upper bound (organisms in ecology, biomolecules in cellular biology)

\[ S = \mathbb{Z}^d_{\geq 0} \]

For \( d \) populations

Other wide application for countable state spaces is spatial lattices

One can also look at discrete-state stochastic processes with continuous time:
Some of the examples described above are more naturally framed in this continuous-time version (biomolecular dynamics, sometimes population dynamics and epidemiology)

Continuous-time Markov chains and renewal processes fall within this framework.

What we will not cover in this class is the case where the state space is uncountably infinite, specifically continuous. \( S = \mathbb{R}^d \)

Doing **continuous state space with discrete time** is actually not difficult but is also not widely used.  

Continuous space, continuous time stochastic processes are often developed in the context of **stochastic differential equations**.