Linear Algebra, MATH–410 Gregor Kovačič

Homework #9 Due: Wednesday, November 3, 1999

<u>All these problems are extra credit.</u>

HINT: In problems 3 and 4, the arithmetic is overwhelming. Help yourself with Maple.

1. Consider the matrix

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ -1 & 0 & 2 \end{pmatrix}.$$

(a) Compute its eigenvalues, and an orthonormal basis in which A will be diagonal.

(b) Show that A is positive definite.

2. Show that the matrices

$$B = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & -1 & -1 \end{pmatrix} \qquad \text{and} \qquad C = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$

commute. Deduce that B and C can therefore be simultaneously diagonalized, and find their eigenvalues, as well as a common orthonormal eigenbasis.

3. Find the eigenvalues and eigenvectors of the orthogonal matrix

$$\mathcal{O} = \frac{1}{9}\begin{pmatrix} 1 & 4 & 8 \\ 8 & -4 & 1 \\ -4 & -7 & 4 \end{pmatrix}.$$

Then find an orthonormal basis in which the matrix $\mathcal{O}$ will assume the real canonical form

$$\begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix},$$

and also the matrix elements of this form.

4. For the matrix

$$M = \begin{pmatrix} -2/3 & 16/9 & 3 \\ 5/3 & -7/9 & -1 \\ -4/3 & -10/9 & 1 \end{pmatrix},$$

Find the positive-definite matrix H and the orthogonal matrix U such that $M = UH$.