Agreement Dynamics and Naming Games in Spatially-Embedded and Random Networks

Q. Lu, G. Korniss (Physics, Rensselaer)
B. Szymanski (Computer Science, Rensselaer)

Supported in part by NSF-DMR and Rensselaer

http://www.rpi.edu/~korniss/Research/
“Low Degree” of Separation

“To prove that nowadays the population of the Earth is in every aspect much more closely interconnected, than it has ever been, one member of our gathering proposed a test. Let us pick at will any given existing person from among the one and one half billion inhabitants of the Earth, at any location. Then our friend bet that he could establish via direct personal links a connection to that person through at most five other persons, one of them being his personal acquaintance. 'As people would say, look, you know X.Y., please tell him to tell Z.V., who is his acquaintance....so and so. …”

“Chain of links” in Everything Is the Other Way by Frigyes Karinthy (1929).
Small-World Networks

- Frigyes Karinthy (1929)
- Stanley Milgram (1967)
- Pool & Kochen (1978).
- Guare (1990)
  “Six Degrees of Separation”
- Watts & Strogatz (1998):
  “... enhanced signal-propagation speed, computational power, and synchronizability”.

 rewiring with probability $p$

\[ p = 0 \quad \text{Increasing randomness} \quad p = 1 \]
Networks & Dynamics on Networks

- Structure of networks
  (prototypical models: Erdős-Rényi ‘60, Watts-Strogatz ‘98, Barabási-Albert ‘99, …)
- Dynamics and interacting particle-systems on static networks (Scalettar 1991, …)
- Co-evolving network structures – coupled to the dynamics on the network
Collective dynamics on the network

Examples:
- Internet (packet traffic/flux)
- Load-balancing schemes (job allocation among processors)
- Electric power grid (voltage and phase fluctuations)
- High-performance or grid-computing networks (task completion landscapes in distributed simulations)
- Synchronization of coupled nonlinear oscillators (phase or frequency)
- Spread of epidemics in cities, human contact networks, or worldwide airline transportation networks
- Dissemination of culture and language in social networks
Models for opinion/agreement dynamics

Models with binary or multiple “opinions” with no threshold:
- kinetic Ising model (Glauber, ’63, …, Castellano et al. ‘05)
- voter model (Krapivsky, ‘92, … Ben-Naim, Redner)
- Naming Game (Baronchelli et al. ‘05, Lu et al. ‘06)

Models with many (discrete or continuous) opinions and with explicit or implicit thresholds:
- individuals too far away on some scale of opinions (or cultural traits) will never interact or convince one another.
- continuous opinion dynamics (Deffuant et al. ‘00, Stauffer et al.‘03, Ben-Naim ‘05)
- dissemination of culture with discrete number of cultural features and traits (Axelrod, 1997, Castellano et al. ’00, Klemm et al. ’03,)
threshold-limited agreement dynamics

individuals $i$ and $j$ interact only if: $|Q_i - Q_j| \leq T$

$N$ individuals

$T > T_c : S_{\text{max}} \sim N$
(size of largest uniform connected cluster)

“global order”
threshold-limited agreement dynamics

\[ Q = \lim_{t \to \infty} \frac{\max S}{N} \]

\[ T < T_c : \quad S_{\text{max}} / N \ll 1 \quad \text{“polarized or multicultural”} \]
Phase diagram of the Axelrod model on regular and small-world networks

$F$ features, $q$ traits for each feature: \( (\sigma_{i1}, \sigma_{i2}, \sigma_{i3}, \ldots, \sigma_{iF}) \)

$\sigma_{ij}, 1, 2, \ldots, q$

$N$ individuals, $i=1, 2, \ldots, N$

Two individuals interact only if they have at least one feature in common

Time evolution in the multicultural regime

San Miguel et al. [Comp. in Sci. and Eng. (IEEE&AIP, 2005)]
Common features of models with threshold

Common features for models with threshold:
[spatio-temporal behavior is much more complex (Redner, ’02)]

Steady-state (equilibrium) phase diagram:
There exists a critical threshold, above which global consensus is reached and prevails.

- Small-world networks: the presence of random links substantially decreases this critical threshold so the region of global consensus expands

- Scale-free networks: for finite networks, there is a system size-dependent critical threshold, but this threshold goes to zero as $N \to \infty$. For larger and larger networks, the region of global consensus progressively dominates the steady-state phase diagram.
Language Games, Semiotic Dynamics

- Evolution of human language

- Artificial and autonomous software agents or robots bootstrapping a shared lexicon without human intervention (Steels, 1995)

- Collaborative tagging: human web users spontaneously create loose categorization schemes ("folksonomy"). See, e.g., del.icio.us and www.flickr.com (Golder & Huberman, ‘05; Cattuto et al., ‘05)
The “Talking Heads” Experiment (the Guessing Game)

http://talking-heads.csl.sony.fr/

“The artificial agents start with no prior human-supplied set of categories nor lexicon. A shared ontology and lexicon must emerge from scratch in a self-organized process.”

(L. Steels, 1995, …)
Naming Game (for a single object)

Temporal Behavior in NG

Baronchelli et al. (2005)
Coarsening in $d$ dimension, $N$ individuals

single lengths scale: (domain/cluster size)

number of different words:

$N_{\text{diff}} \sim \frac{N}{\xi^d(t)} \sim N t^{-d \gamma}$

$N_w$: total number of words:

$N_w - N \sim \frac{N}{\xi^d(t)} \xi^{d-1}(t) \sim \frac{N}{\xi(t)} \sim N t^{-\gamma}$

$S(t)$: success rate of communications:

$1 - S(t) \sim \frac{\xi^{d-1}(t)}{\xi^d(t)} \sim t^{-\gamma}$

$t_c^{-\gamma} \sim N^{1/d}$

$t_c \sim N^{1/(d \gamma)}$

$(d = 1: \gamma = 1/2)$
Language Games in Sensor Networks
Language Games in Sensor Networks

mobile or static sensor nodes deployed in large spatial regions

• environment is unknown, possibly hostile
• tasks are unforeseeable
• sensor nodes have no pre-constructed vocabularies

must autonomously develop common shared vocabularies/language at the exploration stage
Naming Game on Random Geometric Graphs (RGG)

2d RGG

Random geometric networks (above the percolation threshold) *(spatial and random)*
- nodes are connected if they fall within each other’s radio range
- communications: *broadcast* to local neighbors
Naming Game on Random Geometric Networks

(a) 

(b) 

(c) 

(d) 

$t = \infty$
Temporal Scaling in NG on 2d RGG

(a) $N_w(t)/N - 1$

(b) $N_d(t)/N$

(c) $1-S(t)$

$\bar{t}_c \sim N^{1.10}$
Naming Game on Small-World-like RGGs

2d SW-RGG,
density of random links: $p$

\[ \xi(t) \sim t^\gamma \]
\[ l_{SW} \sim p^{-1/d} \]
\[ t_x \sim p^{-1/(d\gamma)} \]
Temporal Behavior in NG on SW-RGG

(2d) RGG: $\bar{t}_c \sim N^{1.10}$

(2d) SW – RGG: $\bar{t}_c \sim N^{0.31}$

$FC (MF): \bar{t}_c \sim N^{0.50}$
Finite-Size Scaling for the agreement time in NG on SW-RGG

\[ t_c = \frac{N^{\alpha_{SW}}}{p^s} f(Np) \]

\[ t_c = \frac{1}{p^{\alpha_{RGG}}} g(Np) \]

\[ g(x) = x^{\alpha_{SW}} f(x) \]

\[ \alpha_{SW} + s = \alpha_{RGG} \]
average consensus times

\[ \bar{t}_c \]

- \( d = 1: \quad \bar{t}_c \sim N^2 \)
- \( d = 2: \quad \bar{t}_c \sim N^{1.10 (1.30)} \)

\textit{d-dimensional coarsening:}

- \( d < d^*: \quad \bar{t}_c \sim N^{1/d\gamma} \)
- \( d > d^* \approx 4: \quad d = \infty (FC): \quad \bar{t}_c \sim N^{0.5} \)

\[ \bar{t}_c \sim \Delta t_c \sim N^\alpha \]

average \quad \text{standard deviation}
Temporal scales in NG on networks

Naming Game in small-world and scale-free networks:

**Optimal trade-off** between FC and behavior:

- **Ordering process is relatively fast**, as in FC network (effectively mean-field dynamics)
- **Memory requirement is small**, as in spatial networks (sparse networks with finite average degree)

<table>
<thead>
<tr>
<th></th>
<th>1d (RGG or regular)</th>
<th>2d (complete graph)</th>
<th>FC (on 1d or 2d RGG or regular)</th>
<th>SW (BA)</th>
<th>SF (BA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>memory need per agent</td>
<td>const.</td>
<td>const.</td>
<td>$N^{0.5}$</td>
<td>const.</td>
<td>const.</td>
</tr>
<tr>
<td>$N_{w}^{\text{max}}/N$</td>
<td>$N^2$</td>
<td>$N^{1.10(1.30)}$</td>
<td>$N^{0.5}$</td>
<td>$N^{0.31(0.42)}$</td>
<td>$N^{0.40}$</td>
</tr>
<tr>
<td>consensus time</td>
<td>$\tilde{t}_c$</td>
<td>$\tilde{t}_c$</td>
<td>$\tilde{t}_c$</td>
<td>$\tilde{t}_c$</td>
<td>$\tilde{t}_c$</td>
</tr>
</tbody>
</table>

(Dall’Asta et al.’06)

- $P(k) \sim 1/k^3$
- $2 < \gamma < 3$
Summary

- Popular [SW (Watts-Strogatz), SF (Barabási-Albert)] network ensembles are “too nice” (strong self-averaging properties): any single realization almost always resembles a typical one, as opposed to “atypical” but sometimes real-life network topologies.

- Many equilibrium and dynamic models on these networks display mean-field features.

A more challenging scenario: networks with community structures (Dall’Asta et al.’06)