Problem 1: Find the fixed points of the following system and classify their stability. Draw a plausible phase portrait.

\[ \dot{x} = xy - x \]
\[ \dot{y} = xy - 3y. \]
**Problem 2:** Find the fixed points of the following system and classify their stability. Draw a plausible phase portrait. What is the equation for the homoclinic orbit with endpoints at the origin (the separatrix)?

\[
\begin{align*}
\dot{x} &= y - y^2, \\
\dot{y} &= x.
\end{align*}
\]
Problem 3: Classify the fixed point of the following system at the origin and show that the system has a periodic solution.

\[
\begin{align*}
\dot{x} &= x + y - x(x^2 + 2y^2), \\
\dot{y} &= -x + y - y(x^2 + 2y^2).
\end{align*}
\]
Problem 4: Solve the following systems for \( x(t) \).

a) \( \ddot{x} - 2\dot{x} - 3x = 9t, \quad x(0) = 0, \quad \dot{x}(0) = -1. \)

b) \( \ddot{x} - 2\dot{x} - 3x = 5e^{-2t}, \quad x(0) = 0, \quad \dot{x}(0) = -1. \)

c) \( \ddot{x} - 2\dot{x} - 3x = 9t + 5e^{-3t}, \quad x(0) = 0, \quad \dot{x}(0) = -2. \)
Problem 5: Find an approximate solution to the following system up to order $O(\epsilon)$ using perturbation theory.

\[
\begin{align*}
\dot{x} + 2x &= -\sin(\epsilon x), \\
x(0) &= 1.
\end{align*}
\]