1) 
\[ x^2y'' + 4xy' + 2y = 0 \]
\[ y = x^r \Rightarrow r(r-1) + 4r + 2 = 0 \]
\[ r^2 + 3r + 2 = 0 \]
\[ (r+1)(r+2) = 0 \]
\[ r_1 = -1, \quad r_2 = -2 \]
\[ y = c_1/x + c_2/x^2 \]

2) 
\[ (x-1)^2y'' + 3(x-1)y' + 12y = 0 \]
\[ y = (x-1)^r \Rightarrow r(r-1) + 3r + 12 = 0 \]
\[ r^2 + 4r + 12 = 0 \]
\[ (r+2)(r+6) = 0 \]
\[ r_1 = -2, \quad r_2 = -6 \]
\[ y = c_1/(x-1)^2 + c_2/(x-1)^6 \]
3) \[ 2x^2y'' - 4xy' + 6y = 0 \]
\[ y = x^r \]
\[ 2r(r-1) - 4r + 6 = 0 \]
\[ r(r-1) - 2r + 3 = 0 \]
\[ r^2 - 3r + 3 = 0 \]
\[ r = \frac{3 \pm \sqrt{9 - 4}}{2} = \frac{3 \pm \sqrt{3}}{2} \]
\[ y = c_1|x|^{3/2} \cos \left( \frac{\sqrt{3}}{2} \ln |x| \right) + c_2|x|^{3/2} \sin \left( \frac{\sqrt{3}}{2} \ln |x| \right) \]

4) \[ x^2y'' - 5xy' + 9y = 0 \]
\[ y = x^r \]
\[ r(r-1) - 5r + 9 = 0 \]
\[ r^2 - 6r + 9 = 0 \]
\[ (r - 3)^2 = 0 \]
\[ y = c_1x^3 + c_2x^3 \ln |x| \]
5) \[ 4x^2 y'' + x^2 y' + 17y = 0 \quad y(1) = 2, \quad y'(1) = -3 \]

\[ y = x^r \quad 4r(r-1) + 2r + 17 = 0 \]
\[ r^2 - r + \frac{17}{4} = 0 \]
\[ r + \frac{17}{4} = 0 \]
\[ r = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{17}{4}} = -\frac{1}{2} \pm 2 \]

\[ y = c_1 \frac{\cos(2 \ln x)}{\sqrt{x}} + c_2 \frac{\sin(2 \ln x)}{\sqrt{x}} \]

\[ y(1) = c_1 = 2 \]

\[ y' = 2 \left[ -2 \frac{\sin(2 \ln x)}{2 \sqrt{x}} - \frac{1}{2} \frac{\cos(2 \ln x)}{\sqrt{x}} \right] \]
\[ + c_2 \left[ -2 \frac{\cos(2 \ln x)}{2 \sqrt{x}} - \frac{1}{2} \frac{\sin(2 \ln x)}{\sqrt{x}} \right] \]

\[ y'(1) = -1 + 2c_2 = -3 \implies c_2 = -1 \]

\[ y = \frac{2 \cos(2 \ln x)}{\sqrt{x}} - \frac{\sin(2 \ln x)}{\sqrt{x}} \]