Suggested Homework 3

In each of the following problems find the general solution of the given differential equation:

1. \( y'' + 2y' - 3y = 0 \)  
2. \( y'' + 5y' = 0 \)  
3. \( y'' - 2y' - 2y = 0 \)

4. Find the solution of the initial value problem

\( y'' + 8y' - 9y = 0, \quad y(0) = 1, \quad y'(0) = 0 \)

Sketch the graph of this solution and describe its behavior as \( x \) increases.

In each of the following problems find the general solution of the given differential equation:

5. \( y'' - 2y' + 6y = 0 \)  
6. \( y'' + 2y' + 2y = 0 \)  
7. \( y'' + 6y' + 13y = 0 \)

8. Find the solution of the initial value problem

\( y'' + 4y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0 \)

Sketch the graph of this solution and describe its behavior as \( x \) increases.

In each of the following two problems find the general solution of the given differential equation:

9. \( y'' - 2y' + y = 0 \)
10. \( y'' - 6y' + 9y = 0 \)

Find the solution of the following initial value problems:

11. \( y'' + y' - 2y = 2x, \quad y(0) = 0, \quad y'(0) = 1 \)
12. \( y'' + 4y = x^2 + 3e^x, \quad y(0) = 0, \quad y'(0) = 2 \)
13. \( y'' - 2y' + y = xe^x + 4, \quad y(0) = 1, \quad y'(0) = 1 \)

In the following problems, determine the suitable form for the particular solution. You do not need to evaluate the constants.

14. \( y'' + 3y' = 2x^4 + x^2e^{-x} + \sin 3x \)
15. \( y'' + 2y' + 2y = 3e^{-x}\cos x + 4e^{-x}x^2\sin x \)
16. \( y'' - 4y' + 4y = 2x^2 + 4xe^{2x} + x\sin 2x \)

In the following three problems, find the general solution to the given differential equation:

17. \( y'' + 2y' = 3 + 4\sin 2x \)
18. \( 2y'' + 3y' + y = x^2 + 3\sin x \)
19. \( \ddot{y} + \omega_0^2 y = \cos \omega_0 t \)

In the following problems, determine the suitable form for the particular solution. You do not need to evaluate the constants.

20. \( y'' + y = x(1 + \sin x) \)
21. \( y'' - 5y' + 6y = e^x \cos 2x + e^{2x}(3x + 4)\sin x \).