Suggested Homework 1

For each of the following differential equations, find the order of the equation and classify it as ordinary (ODE) or partial (PDE) differential equation.

1. \( y' = x + y \)
2. \( xy'' + y = \sin x \)
3. \( u_x = u_{yy} \)
4. \( \frac{\partial^3 y}{\partial x^3} + \frac{\partial^2 y}{\partial z^2} = y \)

Find the general solutions of the following first-order linear differential equations:

5. \( y' + y = xe^{-x} + 1 \)
6. \( xy' + 2y = \sin x \)
7. \( y' + 2xy = 2xe^{-x^2} \)
8. \( (1 + x^2)y' + 4xy = \frac{1}{(1 + x^2)^2} \)

Find the solutions of the following problems:

9. \( y' + \frac{2}{x} y = \frac{\cos x}{x^2}, \quad y(\pi) = 0 \)
10. \( x^2y' + 3xy = \frac{\sin x}{x}, \quad y(\pi) = 0 \)
11. \( xy' + y = e^x, \quad y(1) = 1 \)
12. \( y' + y = \frac{1}{1 + x^2}, \quad y(0) = 0 \)

13. Find the general solution of the equation

\[ y' - \frac{1}{x} y = x \]

What happens to all the solutions as \( x \to 0? \)