Suggested Homework 7

Section 10.6.

3. Show that the power series (a)-(c) have the same radius of convergence. Then show that (a) diverges at both endpoints, (b) converges at one endpoint but diverges at the other, and (c) diverges at both endpoints.

(a) $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

(b) $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

(c) $\sum_{n=1}^{\infty} \frac{x^n}{n^23^n}$

5. Show that $\sum_{n=0}^{\infty} n^n x^n$ diverges for all $x \neq 0$. Find the interval of convergence.

9. $\sum_{n=0}^{\infty} nx^n$

10. $\sum_{n=1}^{\infty} \frac{2^n}{n} x^n$

13. $\sum_{n=4}^{\infty} \frac{x^n}{n^5}$

15. $\sum_{n=0}^{\infty} \frac{x^n}{(n!)^2}$

16. $\sum_{n=0}^{\infty} \frac{8^n}{n!} x^n$

25. $\sum_{n=1}^{\infty} n(x - 3)^n$

26. $\sum_{n=1}^{\infty} \frac{(-5)^n(x - 3)^n}{n^2}$

29. $\sum_{n=1}^{\infty} \frac{2^n}{3n} (x + 3)^n$

30. $\sum_{n=0}^{\infty} \frac{(-5)^n}{n!} (x + 10)^n$

32. $\sum_{n=10}^{\infty} n!(x + 5)^n$

Use equation (2)

$$\frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n \quad \text{for} \quad |x| < 1$$

to expand the function in a power series with center $c = 0$ and determine the interval of convergence.

35. $f(x) = \frac{1}{1 - 3x}$

37. $f(x) = \frac{1}{3 - x}$

38. $f(x) = \frac{1}{4 + 3x}$

39. $f(x) = \frac{1}{1 + x^2}$
41. Use the equalities

\[
\frac{1}{1 - x} = \frac{1}{-3 - (x - 4)} = \frac{-\frac{1}{3}}{1 + \left(\frac{x - 4}{3}\right)}
\]

to show that for \(|x - 4| < 3\),

\[
\frac{1}{1 - x} = \sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x - 4)^n}{3^{n+1}}
\]

42. Use the method of Exercise 41 to expand \(1/(1 - x)\) in power series with centers \(c = 2\) and \(c = -2\). Determine the interval of convergence.

43. Use the method of Exercise 41 to expand \(1/(4 - x)\) in a power series with center \(c = 5\). Determine the interval of convergence.