

## Tridiagonal Solver

To solve  $\mathbf{A}\mathbf{y} = \mathbf{f}$  where  $\mathbf{A}$  is an  $N \times N$  tridiagonal matrix of the form

$$\mathbf{A} = \begin{array}{ccccccc} & & a_1 & c_1 & & & \\ & & b_2 & a_2 & c_2 & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & \cdot & \cdot & \cdot \\ & & & & & b_{N-1} & a_{N-1} & c_{N-1} \\ & & & & & & b_N & a_N \end{array}$$

the following algorithm can be used:

*Set:*

$$w_1 = a_1 \quad v_1 = c_1/w_1 \quad z_1 = f_1/w_1$$

*For*  $i = 2, 3, \dots, N$

$$w_i = a_i - b_i v_{i-1}$$

$$v_i = c_i/w_i$$

$$z_i = (f_i - b_i z_{i-1})/w_i$$

*Set:*

$$y_N = z_N$$

*For*  $j = N - 1, N - 2, \dots, 1$

$$y_j = z_j - v_j y_{j+1}$$

Note the storage for this algorithm requires five  $N$ -vectors ( $\mathbf{y}$ ,  $\mathbf{f}$ ,  $\mathbf{w}$ ,  $\mathbf{v}$ ,  $\mathbf{z}$ ) and the operation count is  $8N - 6$  flops.