General note: For computer exercises please provide listings of all computer programs that were used to generate the results along with the output and plots.

1. (10 pts) Read chapter 0 in the book. Also read Appendix B (Introduction to Matlab) in the book. If you are new to Matlab, type in the commands from Appendix B for practice or type in the commands from \texttt{matlabIntro.m} available from the class web page.

   To get full marks for this problem you should state that you have actually done the readings and you have either practiced with Matlab or that you are already an expert.

2. (20 pts) Implement the \texttt{nest} function (from chapter 0) for nested evaluation of a polynomial (Horner’s rule) in Matlab. Use the \texttt{nest} function to evaluate the following polynomial (Taylor series) approximation to $e^x$,

   $$p(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3,$$

   at the point $x = 1$ and confirm that the answer matches the answer you get by evaluating $p(1)$ directly using Matlab. Print the result from \texttt{nest} and Matlab and the difference. Also print $e^1$ for comparison.

   Evaluate and plot both the polynomial $y = p(x)$ and $y_e = e^x$ at $N$ equally spaced points on the interval $[a, b]$,

   $$x_i = a + (b - a) \frac{(i - 1)}{(N - 1)}, \quad i = 1, 2, \ldots, N.$$

   - Choose $N = 21$ and $[a, b] = [0, 2]$.
   - Plot the curves with different colors and different markers (e.g. draw $p(x)$ as a red curve with an “x” at each grid point).
   - Label the $x$ and $y$ axes.
   - Use a legend to label the curves.
   - Give the plot the title \textit{Results from function nest}.

3. (10 pts) Find the Taylor series polynomial of degree 3, plus remainder term, $f(x) = p(x) + R(x)$, about the point $x = 0$ for the following functions $f(x)$. The error is $e(x) = f(x) - p(x) = R(x)$. From the remainder term, $R(x)$, determine an upper bound on the absolute value of the error at $x = 0.1$. Compute the actual error using a calculator (or Matlab) and confirm your upper bound.

   (a) $f(x) = \sin(2x)$

   (b) $f(x) = \sqrt{1 + x}$