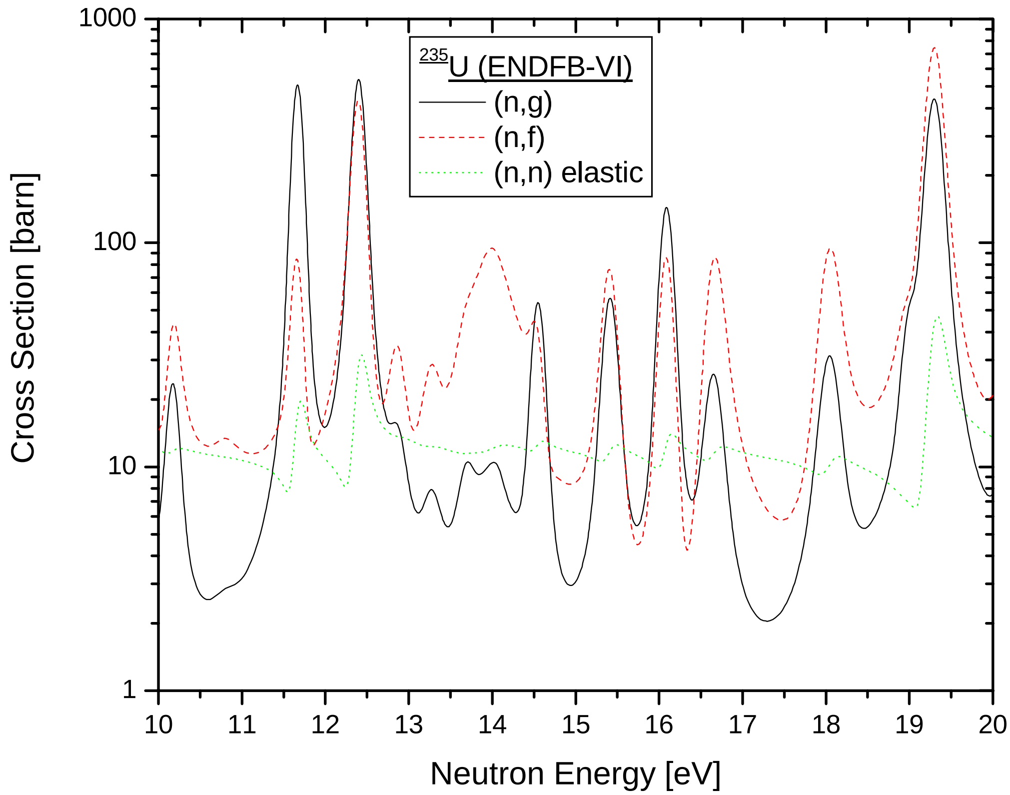

Figure 5 – neutron capture, fission and elastic scattering in ^{235}U.

For spaced resonances the center of mass (CM) energy E_c dependence of the neutron induced reaction cross section is given by the Breit-Wigner Single-level resonance formula[1].

$$\sigma_x(E_c) = \pi\lambda\!\!\!^{-2} g \frac{\Gamma_n \Gamma_x}{(E_c - E_0)^2 + \frac{1}{4}\Gamma^2} \tag{1.20}$$

Where E_0 is the resonance energy in the CM system. The relation between the center of mass energy E_c to the laboratory energy E_L is given by :

$$E_c = \frac{A}{A+1} E_L, \tag{1.21}$$

Notice that for heavy nuclei $E_c \cong E_L$. The reduced neutron wavelength, $\lambda\!\!\!^{-}$ is given by:

$$\lambda\!\!\!^{-} = \frac{\hbar}{\sqrt{\frac{Mm}{M+m} 2E_c}}, \tag{1.22}$$

[1] Bell and Glasstone, *Nuclear Reactor Theory*, p. 392, 1970.

Where *m* is the neutron mass and *M* is the target nucleus mass. The reduced neutron wavelength can also be written in the form:

$$\lambda\!\!\!^{-2} = \lambda\!\!\!^{-2}_0 \frac{E_0}{E_c} \tag{1.23}$$

Where $\lambda\!\!\!^{-}_0$ is given by:

$$\lambda\!\!\!^{-2}_0 = \lambda^2(E_0) = \frac{\hbar^2}{\frac{Mm}{M+m}2E_0} \cong \frac{A+1}{Am}\frac{\hbar^2}{2E_0} \tag{1.24}$$

The factor *g* is a statistical factor that measures the probability that a particular compound state will form.

$$g = \frac{2J+1}{2(2I+1)}\,. \tag{1.25}$$

Where is *I* the spin of the target nuclei, for a given neutron orbital angular momentum *l*. The compound nucleus spin *J* is bound by:

$$\left|I - l \pm \tfrac{1}{2}\right| \le J \le \left|I + l + \tfrac{1}{2}\right|, \tag{1.26}$$

At low neutrons energies, resonances are likely to be s-wave *l*=0 and we get.

$$g = \frac{1}{2}\left(1 \pm \frac{1}{2I+1}\right) \tag{1.27}$$

The widths are noted by the Greek letter Γ. They measure the probability that the compound nucleus will decay within a specific time. The width can also be written as $\Gamma_x = \hbar / \tau_x$ where τ_x is the decay constant of the compound nucleus.

The neutron width Γ_n has an energy dependence of the form:

$$\Gamma_n(E_c) = \Gamma_n(E_0)\sqrt{\frac{E_c}{E_0}}\,, \tag{1.28}$$

Total width is calculated at (E_0)

$$\Gamma = \Gamma_n(E_0) + \sum_x \Gamma_x \qquad (1.29$$

Substituting equations 1.23 and 1.28 to equation 1.20 we get

$$\sigma_x(E_c) = \pi\lambda_0^2 \frac{E_0}{E_c} g \frac{\Gamma_n(E_0)\sqrt{\frac{E_c}{E_0}}\Gamma_x}{(E_c - E_0)^2 + \frac{1}{4}\Gamma^2} = 4\pi\lambda_0^2 g \sqrt{\frac{E_0}{E_c}} \frac{\Gamma_n(E_0)\Gamma_x}{4(E_c - E_0)^2 + \Gamma^2}, \qquad (1.30)$$

this expression can be rewritten as;

$$\sigma_x(E_c) = 4\pi\lambda_0^2 g \frac{\Gamma_n(E_0)}{\Gamma} \frac{\Gamma_x}{\Gamma} \sqrt{\frac{E_0}{E_c}} \frac{\Gamma^2}{4(E_c - E_0)^2 + \Gamma^2}. \qquad (1.31)$$

Where x could be γ for the capture cross-section or f for fission. Rearranging we can write:

$$\boxed{\sigma_x(E_c) = \sigma_0 \frac{\Gamma_x}{\Gamma} \sqrt{\frac{E_0}{E_c}} \frac{\Gamma^2}{4(E_c - E_0)^2 + \Gamma^2}}, \qquad (1.32)$$

where;

$$\sigma_0 = 4\pi\lambda_0^2 g \frac{\Gamma_n(E_0)}{\Gamma} = 4\pi \frac{A+1}{Am} \frac{\hbar^2}{2E_0} g \frac{\Gamma_n(E_0)}{\Gamma} = \frac{4\pi\hbar^2}{2m} \frac{A+1}{E_0 A} \frac{g\Gamma_n(E_0)}{\Gamma}, \qquad (1.33)$$

evaluating the constants we get;

$$\boxed{\sigma_0 = \frac{2.608x10^6}{E_0} \frac{A+1}{A} \frac{g\Gamma_n(E_0)}{\Gamma}}. \qquad (1.34)$$

When E_0 and E are in units of eV then σ_0 is in barns. The value of the cross section at the peak of the resonance is given by $\sigma_0 \frac{\Gamma_x}{\Gamma}$ this can be easily tested by setting E_c=E_0 in equation 1.32.

The elastic neutron scattering cross section for l=0 (low energy neutrons) is given by

$$\sigma_s(E_c) = \sigma_0 \sqrt{\frac{E_0}{E}} \frac{\Gamma^2}{4(E - E_0)^2 + \Gamma^2} \left[\frac{\Gamma_n}{\Gamma} + \frac{4(E - E_0)}{\Gamma} \frac{R}{\lambda} \right] + 4\pi R^2 \qquad (1.35)$$

Where R is effective radius of the nucleus ($R \approx 1.25 \times 10^{-13} A^{\frac{1}{3}}$ cm). The last term in equation 1.35 is the potential scattering and it represents the scattering cross section between the resonances. In light nuclei the potential scattering is dominant.

Example

Calculate the scattering cross section at the peak of the 36.67 eV resonance in ^{238}U.

At the peak of the resonance, the scattering cross section is $\sigma_s = \sigma_0 \frac{\Gamma_n}{\Gamma} + 4\pi R^2$ we can get all the resonance parameters from http://www.nndc.bnl.gov/nndc/endf/.

Γ_γ=2.28900 x10^{-2} eV

Γ_n=3.413000x10^{-2} eV

J=0.5, I=0 then g=1

$$\sigma_0 = \frac{2.608x10^6}{E_0}\frac{A+1}{A}\frac{g\Gamma_n(E_0)}{\Gamma} = \frac{2.608x10^6}{36.67}\frac{239}{238}\frac{0.03413}{0.03413+0.02289} = 42749.1\,\text{barn}$$

The peak scattering cross section is then given by

$$\sigma_s(36.67) = 42749.1\frac{0.03413}{0.03413+0.02289} + 4\pi\left(1.25\times10^{-13}238^{\frac{1}{3}}\right)^2\times10^{24} = 25588 + 7.5\,\text{barn}$$

This is a very large cross-section.