Dead time and pileup in pulsed parametric X-ray spectroscopy

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Received 28 August 2003; received in revised form 8 December 2003; accepted 21 December 2003

Abstract

Spectroscopy of parametric X-rays (PXR) from pulsed electron sources is usually done with very low electron beam currents in order to avoid dead time losses and pileup problems. This paper presents a method that allows correction of dead time losses and pileup in pulsed X-ray spectroscopy. Such correction is required in order to study the variation of PXR production and linewidth with increased electron beam currents. The method presented here uses the observed integrated count rate in a PXR peak and the total count rate to obtain the dead time and pileup corrected rates. The method also allows treatment of pulse pileup from a first-order reflection to a second-order reflection and can be extended to higher order reflections. Application of the theory to measured data is demonstrated and corrections of up to a factor of four are demonstrated. The method described in this paper can be applied to any pulsed spectrum measurement and is not specific to PXR, which was used here for comparison of the theory to experiments.

PACS: 29.30.Kv; 32.30.Rj; 29.25.Bx

Keywords: Parametric X-rays; Dead time; Pileup; Pulsed source

1. Introduction

Pulsed X-rays can be produced using variety of methods; in this paper we will concentrate on sources capable of producing pulses of monoenergetic X-ray photons. One such source is currently being investigated at Rensselaer Polytechnic Institute as a potential source of intense quasi-monochromatic X-rays [1–3]. This source is based on a production mechanism known as parametric X-ray (PXR); numerous experiments to characterize these X-rays and comparisons with theoretical calculations were reported elsewhere, see for example, Refs. [4,5]. Because the production rate of PXR is linear with the electron beam current and in order to minimize dead time losses and pileup effects, all previous experiments were done at low electron currents (4–20 nA) [6]. It is of interest to study the effect of higher electron currents on the PXR production because the high current can cause local heating in the crystal that might distort the spectrum. The problem with such experiments is that spectroscopy is based on counting single pulses and will suffer significant dead time losses at high PXR rates. The dead time is due to the resolving time associated with the detector itself, its amplifier and the time required...
for the ADC to convert the pulse to a digital form used for spectroscopy. During this dead time the system cannot respond to other photons that hit the detector and these events will not be counted and thus are lost. One option to cope with high rates is to lower the detector efficiency, for example, by collimation or absorption of the PXR beam. For the case of absorption, the effect will be mostly on the PXR and less on the higher energy photon background which will result in reduction of the signal-to-background ratio. In such a case, background photons will cause dead time losses in counting PXR events and a model that allows correction for this effect is required. In this paper a model is provided that allows, in conjunction with collimation and absorption, dead time losses and pileup corrections to be made. As will be explained later, for pulsed sources this maximum observed rate is limited to the pulse repetition rate of the electron source.

2. PXR production and measurement

To produce PXR, a relativistic beam of electrons is passed through a crystal and the emitted X-rays satisfy the Bragg (or Laue) condition [7] as illustrated in Fig. 1. The advantages of PXR are high intensity in terms of photons produced per electron, narrow energy band, large emission angle relative to the electron beam direction and narrow emission cone [8]. At RPI we use 60 MeV electrons from the RPI LINAC delivered in a pulse about 30 ns wide and at a repetition rate of 400 pulses per second. In the experiments discussed in this paper, the electron beam passed through a 1 mm thick LiF crystal and PXR was produced from the $<200>$ planes. The Bragg angle $\theta$ was 30°.

![Fig. 1. Typical geometry for PXR production.](image)

A typical spectrum is shown in Fig. 2. Because of the small size of the crystal (1 cm x 1 cm) it typically intercepted about 20% of the electron beam current. The first-order reflection from the (200) plane is not visible because the 6.2 keV photons it produces are absorbed in the air gap between the crystal and the detector; the higher order reflections are clearly visible.

The X-rays shown in Fig. 2 were collected using an Amptek XR-100CR 500 μm thick Si detector with intrinsic photopeak efficiency of about 87% at 12 keV. The detector was positioned 3 m from the crystal behind a 40 cm long lead collimator with a 0.3 cm x 0.3 cm hole; this arrangement defines the detection solid angle to $1 \times 10^{-6}$ sr. The PXR are produced during the electron pulse, which in the case discussed here is much shorter than the resolving time of the detector. The resolving time of the Amptek detector is determined by the amplifier’s shaping time constant, which is 12 μs. In such a case, the detector is able to detect only one event per LINAC pulse, which limits the maximum possible count rate to the repetition rate of the LINAC that was set to 400 pulses per second. This situation can create a pileup problem if two or more photons reach the detector during the pulse. In such a case, the observed energy for this event will be the sum of

![Fig. 2. A typical PXR spectrum measured at an average LINAC beam current of 617 nA and pulse repetition rate of 400 Hz.](image)
the energy deposited by all the photons. The amount of pileup is a function of the count rate and for certain rates can produce peaks that look like high-order X-ray production (reflection) peaks. Another problem that occurs in such an experiment is that background signals can cause significant dead time losses in the detector. The term background refers to photons with energies other than that of the peak of interest. The background photons can have a broad energy spectrum and are a result of Bremsstrahlung produced by the crystal and reaching the detector. Thus the contribution of the background photons to the measured spectrum can be observed in all the energy bins. In the experiment described here, the background pulses from high-energy photons with amplitudes beyond the last channel of the MCA where collected in the last bin of the energy spectrum as shown in Fig. 2. If a background photon and a PXR photon that originated from the same LINAC pulse both reach the detector they will be recorded as one background event, which causes the loss of a valid PXR photon count.

In this paper we will introduce a model for the observed count rate in a peak as a function of the real rate and present a method to correct the observed count rate in a peak for dead time losses introduced by the background photons or by X-ray photons from other reflections. This treatment is very general and not specific to PXR generated X-rays and thus can be applied for similar spectral measurements with pulsed sources.

3. Theory

In order to correct for dead time losses and pileup in the measured X-ray spectrum, a model describing the observed rate in the detector is developed. This correction can be applied to determine the PXR rate that would be observed in a peak if the detector were able to resolve all the pulses emitted during the pulse and there were no dead time losses or pileup.

This derivation requires several assumptions; (a) the detector resolving time is longer than the X-ray pulse width and thus only one event per pulse can be measured, (b) the detector can fully recover and is ready to measure at the beginning of each radiation pulse, which implies that the detector resolving time must be shorter than the beam-off interval and (c) there are no measured background photons or noise in between the radiation pulses.

3.1. Dead time and pileup corrections for the first observed peak

For the analysis, an assumption is made that the processes of PXR and Bremsstrahlung emission follow the Poisson distribution. This is reasonable because the production process is based on interactions which follow Poisson statistics; the attenuation of the photons by air on their way to the detector and the absorption probabilities in the detector will not change the Poisson nature of the process. Thus the observed events will also follow Poisson statistics [9].

Define $m$ as the mean number of photons per X-ray pulse that can be detected by a detector with no dead time losses. Thus, the probability of such a detector to detect $n$ events during a pulse is given by the Poisson probability function:

$$P(n, m) = \frac{m^n e^{-m}}{n!}.$$  \hspace{1cm} (1)

The number of PXR photons with energy corresponding to the first observed peak (peak-1 in Fig. 2) that can be recorded by a detector with no dead time losses during a pulse is noted as $m_1$ and similarly the total number of photons detected per pulse is noted as $m_t$. The background for a detector with no dead time losses is defined as the total number of photons that can be detected minus the number of photons that were detected with energy corresponding to peak-1: $m_b = m_t - m_1$. Using $m_1$ and $m_b$ expressions can be written for the probabilities to observe PXR photons in peak-1 $P_1$, and any photon $P_t$ during a pulse

$$P_1 = P(1, m_1) P(0, m_b)$$ \hspace{1cm} (2)

$$P_t = 1 - P(0, m_t).$$ \hspace{1cm} (3)

The expression in Eq. (2) is the probability to get one PXR photon and no background photons, which is the only possibility to get a PXR photon contributing to peak-1 during an X-ray pulse.
These expressions neglect second-order effects such as two background photons piling up to exactly the energy of a PXR peak, however, the probability for such an event is low even in relatively high counting rates.

The corresponding observed rates \( r_1 \) and \( r_t \) can be calculated by multiplication of Eqs. (2) and (3) by the pulse frequency of the LINAC \( f \), 

\[
r_1 = f P_1 \exp(-m_1) \exp(-m_t + m_1) \\
r_t = f [1 - \exp(-m_1)].
\]

Eqs. (4) and (5) can be solved for the dead time corrected count rates \( n_t \) and \( n_1 \) by noting that these rates are given by multiplying the no dead time counts per pulse \( m_t \) and \( m_1 \) by the LINAC frequency \( n_t = fm_t \) and \( n_1 = fm_1 \)

\[
n_t = f \ln \left( \frac{f}{f - r_t} \right)
\]

and

\[
n_1 = \frac{f r_1}{f - r_1}
\]

These simple expressions allow to correct the observed rates \( r_1 \) and \( r_t \) for the dead time losses associated with the pulsed nature of this measurement. The derived expressions do not depend on the X-ray pulse width or the detector resolving time.

Eq. (6) for the total count rate is a well-known expression for a pulsed source for the case when the detector resolving time is larger than the pulse width [10,11]. A similar derivation for the total rate with different detector resolving time models was also given by Sunji Kishimoto [12] and was also mentioned in Ref. [9]. The correction factor for the peak depends only on the pulse repetition rate \( f \) and the observed total rate; this is illustrated in Fig. 3. This figure also shows that at high total rates the correction factor of the peak becomes very large, for example, at a total rate of 200 cps the correction factor for a peak is 2 and at 300 cps it is doubled to 4.

3.2. Dead time and pileup corrections for a higher order peak

Similar expressions can be written for higher order peaks. For example, peak-3 in Fig. 2, can have contributions from direct PXR production at that energy and from pileup of two photons with energy corresponding to peak-1. Contributions can also come from a combination of peak-1 energy photons and background X-ray that corresponds exactly to the same energy, however the probability for this event is low and it was neglected. The expression for the probability to get a photon counted in peak-3 is given by

\[
P_2 = P(0,m_b)[P(2,m_1)P(0,m_2)] + P(0,m_1)P(1,m_2)
\]

where in this case the background counts are given by \( m_b = m_t - m_1 - m_2 \).

In this case we are solving for \( n_2 \), which is the dead time corrected count rate; also we note as in previous cases that \( r_2 = f P_2 \). After some manipulation, Eq. (8) can be explicitly written for the observed rate \( r_2 \)

\[
r_2 = f \exp(-m_t) \left( \frac{1}{2} m_1^2 + m_2 \right).
\]
Using Eqs. (5) and (7), substitutions for \( \exp(-m_t) \) and \( m_1 \) in Eq. (9) can be obtained
\[
\exp(-m_t) = \frac{f - r_1}{f} \tag{10}
\]
\[
m_1 = \frac{r_1}{f - r_1} \tag{11}
\]

The expressions in Eqs. (10) and (11) can now be substituted in Eq. (9), and solution for \( n_2 \) can be readily obtained
\[
n_2 = f \frac{r_2}{f - r_1} - \frac{1}{2} \left( \frac{r_1}{f - r_1} \right)^2 \tag{12}
\]

Eq. (12) provides a correction for both the dead time losses and pileup for the counts in peak-3. The first part of Eq. (12) \( r_2/(f - r_1) \), is the dead time correction, which is similar to the dead time correction that was applied to the first-order reflection (peak-1) in Eq. (7). The second part
\[
\frac{1}{2} \left( \frac{r_1}{f - r_1} \right)^2 = \frac{1}{2} m_1^2
\]
is the pileup correction. This procedure can be generalized for higher order reflections, but this will be not presented here.

4. Experimental verification

Two sets of PXR production experiments in Bragg geometry were done using the experimental setup described in Section 2. Each of the two experiments contains a set of spectra collected for different LINAC electron beam currents. An example of a typical spectrum is shown in Fig. 2. In order to estimate the PXR count rate as a function of electron beam intensity we integrated the area under each peak subtracting the background under the peak, which was treated as a simple straight line. Also the total sum of all the counts in the spectrum (including the counts in the saturation peak) was calculated. This peak represents high-energy photons that saturated the detector electronics. The data were collected for 120 s; the count rates (cps) are plotted in Fig. 4.

Fig. 4 shows that the total count rate is increasing with the beam current and because only one photon can be measured per LINAC pulse, it is bound by the LINAC pulse repetition rate, which was set at 400 Hz in this experiment. The counts under peak-1 seem to reach saturation in experiment 1. In experiment 2, which was done with higher total rate, peak-1 reaches a maximum and then as the current increases the count rate drops. As evident from Fig. 4 the experiments were not done under similar conditions, experiment 2 was done in better conditions and yielded more PXR but also more background per electron.

As mentioned earlier, the first-order reflection that occurs at an energy of 6.2 keV is fully absorbed by the air between the source and the detector (the calculated attenuation factor is \( \sim 2000 \)) and thus the first observed peak is from a second-order reflection. However, since the photons from the first-order reflection do not reach the detector no pileup will exist and the first observed peak (the second-order reflection) could be used to test our dead time correction procedure. In the case of experiment 1 as shown in Fig. 2, peak-3 will have contributions from photons with energy 24.95 keV and also from pileup events of two photons with an energy of peak-1.

In order to test the model, Eqs. (6) and (7) were used to obtain the dead time corrected count rates for each experiment; the dead time corrected rates
are plotted in Fig. 5. As expected, the production rate is linear with the beam current for all four cases except for the last two points of experiment 2, where the observed rate ($r_t$ in Fig. 4) exceeds 300 cps. At the point where the deviation from linearity occurs ($I = 20 \times 10^9$ electrons/pulse) the corrected rate of peak-1 is a factor of 4 higher than the measured rate (Fig. 4), this is rather a large dead time correction. Also, propagation of the statistical error indicates that the error in the measured counts is amplified by the correction. This is illustrated in Fig. 5 for the error in peak-1 of experiment 2, this is a limitation that manifests itself as the total rate gets closer to the LINAC pulse repetition rate $f$.

Using the linear relations of Fig. 5 for the dead time corrected count rate $n_1$ and $n_t$, the rates per pulse $m_1$ and $m_t$ can be calculated and inserted to Eqs. (4) and (5) to calculate the observed rates and compare them with the measurements, the results are plotted in Fig. 6. The comparison in Fig. 6 shows that the calculated rates are in very good agreement with the experimental results. It should be noted that the linear fit used for peak-1 of experiment 2 and shown in Fig. 5 was calculated ignoring the two last points that did not seem linear with the current. However ignoring these two points still yields a reasonable fit as shown in

Fig. 6. The reason for this effect is that for high count rates close to the pulse repetition rate $f$, the observed rates (Eqs. (4) and (5)) are not very sensitive functions of the dead time corrected counts $m_1$ and $m_t$.

As discussed in the previous section, a more complicated problem is encountered when analyzing a higher order reflection. In a case of a second-order reflection, for example, the counts in the peak could be due to PXR at the peak energy and also due to pileup of two photons with the energy of a first-order photon. To demonstrate such a correction, peak-1 and peak-3 of Fig. 2 can be used. Because peak-3 occurs at double the energy of peak-1 it is expected that pileup will occur at that peak. To illustrate the problem, notice that in Fig. 2, peak-2 is larger than peak-3 as expected from the theory of PXR production. As mentioned before it is expected that the production rate will be linear with the beam current, thus with no pileup the ratio of the areas under these peaks will remain constant. Fig. 7 shows the number of counts in peak-2 and peak-3 in experiment 1 as a function of the electron beam current. It is evident that peak-2 reaches saturation at high electron beam currents while peak-3 is still increasing until it surpasses the count rate of peak-2. The reason for this increase is the
contribution of counts from pileup of PXR photons with the energy of peak-1.

To test our model Eq. (12) was used to obtain the dead time losses and pileup corrections of the counts in peak-3. A straight fit to the data resulted in the function $n_3 = 0.15I \times 10^9$, this fit was then inserted back to Eq. (9) together with the previously obtained fits for $m_1$ and $m_2$ as was shown in Fig. 6, and the result is then plotted in Fig. 7 with the measured data for peak-3. The calculation show reasonable agreement with the experiment, the calculations also indicate that as the beam power increases the pileup increases. For example, for the case of $I = 20 \times 10^9$ e/pulse the counts from pileup are calculated to be about 2.2 times higher than the counts from PXR.

5. Conclusions

A simple method for correction of dead time losses and pileup in pulsed X-ray spectroscopy was developed. This method can be used to measure the true count rate in a measured peak by correcting for dead time losses and pileup effects. This model is valid for the case where the detector resolving time is larger than the X-ray burst width; in such a case the total count rate is limited by the pulse repetition rate. Also in order to apply this method to an X-ray peak, the total count rate is required, in most cases this can be easily obtained by summing all the counts in the measured X-ray spectrum. However care should be taken not to have significant amounts of pulses discriminated as in such a case the measured rate is not the actual rate of events in the detector.

The model was applied to pulsed PXR production from a LINAC operated with a pulse repetition rate of 400 pulses/s and high rate measurements of up to 370 cps. The dead time and pileup corrected rates were obtained and behave linearly with the electron beam as expected. For a repetition rate of 400 pulses/s, at total count rates above 300 cps the correction becomes less accurate. When the measurements are done with the presence of background photons during the radiation pulse, the correction factors can be high, in the experiments reported here a correction factor of up to 4 yielded reliable results. The model was also applied to a second-order reflection where pileup from a first-order reflection was about 55% of the measured count rate in the peak.

The method described in this paper can be applied to any pulse spectrum measurements and is not specific to PXR, which was used here for comparison of the theory to experiments.

References


